# SECOND GRADE PERSPECTIVE,

(THEORY AND PRACTICE.)

FIFTEENTH EDITION.

SPECIALLY PREPARED

FOR THE USE OF ART STUDENTS,

BY

H. J. DENNIS,

AUTHOR OF "THIRD GRADE PERSPECTIVE;" ART MASTER, LAMBETH SCHOOL OF ART, DULWICH COLLEGE, STOCKWELL TRAINING COLLEGE, ETC.

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[ENTERED AT STATIONERS' HALL.]

1893.

# PREFACE.

It is probable, that the principal motive of every teacher in compiling a book, is the improvement of his own pupils; such, at least, is the immediate object of the present work, and secondly, as a manual for self-instruction for the public generally.

In writing my former work, THIRD GRADE PERSPECTIVE, I intentionally omitted the Theory, thinking that an elementary work was not needed; however, since its publication large numbers of Art Students and professional friends have convinced me that a work treating fully of the Theory would be an especial boon.

It is highly important that the student should have a knowledge of Geometry previously to commencing the study of Perspective. It is impossible to over-estimate the importance of it, forming, as it does, the basis of the Art of Perspective.

It is very essential that the student should thoroughly understand the Theory, and, as the latter pages of this book owe their perspicuity to those which precede them, he is advised to read and pay particular attention to the whole, and not pass over any portion until the preceding information has been clearly understood.

I am much indebted to Professor Erasmus Wilson, F.R.S., for his great kindness in allowing me to make extracts from his work, "Anatomists' Vade Mecum," of the construction and functions of the human eye.

In introducing a further edition of Second Grade Perspective, I must express my gratification at the success the work has obtained, and the favour with which it has been received both by Art Masters and Students. And, with a view to its further usefulness, copies of the latest Government Examination Papers have been added, which will not only be found valuable, but, I think, will be appreciated by all who are preparing themselves for examination in the subject.

Etruria, Cautley Avenue,
Clapham Common, S.W.
*March*, 1893

Henry J. Dennis
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# LIST OF ABBREVIATIONS.

| | |
|---|---|
| C V.......................................................... | Centre of vision. |
| V L.......................................................... | Vanishing line. |
| *I L .......................................................... | Intersecting line. |
| V P.......................................................... | Vanishing point. |
| M, or M P.................................................. | Measuring point. |
| D 1, D 2 ................................................ | Distance points |
| Van. Par. ................................................ | Vanishing parallel. |
| P V R...................................................... | Principal visual ray. |
| 30° .......................................................... | 30 degrees. |
| 2′ .......................................................... | 2 feet. |
| 2′ 6″ ...................................................... | 2 feet 6 inches. |

The intersections of planes are shown by chain-lines, thus: —— - —— - —— - —— - ——.

The construction lines of all the diagrams are shown and fully explained.

In copying the plates, the student is recommended to use a scale of half-an-inch to a foot.

* It is now called Picture Line.

# CONTENTS.

CONTENTS.

# THEORY OF PERSPECTIVE.

The literal meaning of the term Perspective is *to see through.*

Everything in nature is seen through the transparent medium of the atmosphere. In practice, the objects are viewed through an imaginary transparent plane, as a sheet of glass.

## DEFINITION OF A PLANE.

A plane is an even surface, perfectly free from the least concavity or convexity; it is such, that if any two points lying in it be joined, the straight line will lie wholly in the plane.

A plane has length and breadth, but no thickness; a leaf of this book may serve as an illustration.

There are three kinds of planes.

Horizontal Planes are parallel to the horizon, as the ceiling and floor of a room.

Vertical Planes are upright, perpendicular to the horizon, as the walls of a room.

Oblique, or Inclined Planes, are those which are neither horizontal nor vertical, as the inclined surfaces of the roof of a building.

The intersection of any two planes is a straight line.

*The imaginary planes used in the practice of perspective are rectangular in form, and indefinite in extent.*

## DEFINITION OF A SOLID.

A solid has length, breadth, and thickness, and is composed of points, lines, and planes. Its corners are points, its edges are lines, and its surfaces are planes.

## DEFINITION OF PROJECTION.

Projection is the representation of objects upon a plane by means of straight lines, called rays of light, drawn from every visible part of such objects to intersect the plane of projection.

The principal kinds of projection are *Orthographic* and *Perspective.*

## ORTHOGRAPHIC PROJECTION.

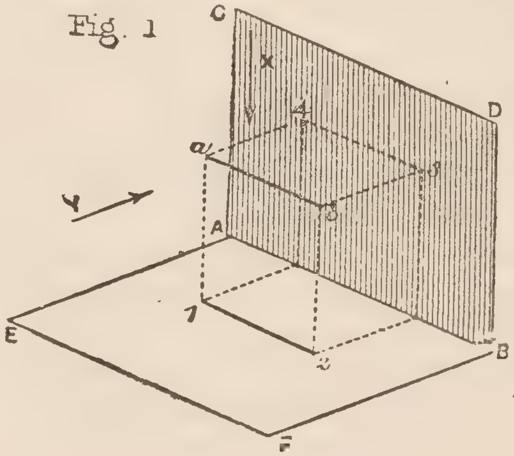
The orthographic projection of a solid must be determined by at least two distinct views: one from immediately above, the other from the front or side.

The view from above is called the *plan*, and that from the front or side is called the *front* or *side elevation*, as the case may be.

In orthographic projection the rays of light are supposed to be parallel to each other, and *perpendicular* to the plane of projection; therefore, when the object is viewed from above, its projection is determined upon a *horizontal plane,* and when viewed from the front or side, its projection is found upon a *vertical plane.*

Let **A B C D**, **A B E F**, Fig. 1, be the vertical and horizontal planes of projection, and imagine **a b** to be a given line in space parallel to the planes of projection, in front of the vertical plane, and above the horizontal plane. Required the plan and elevation of **a b**.

Fig. 1




If we imagine our eyes immediately above **a b**, as indicated by the arrow **X**, its plan will be represented by the line **1 2**, because the parallel rays of light from **a** and **b** meet the plane of projection at points **1** and **2**.

Again, if we imagine our eyes

immediately in front of the given line **a b**, as indicated by the arrow **Y**, its elevation will be the line **3 4**, because the rays of light intersect the vertical plane of projection at the points **3** and **4**.

*N.B.—In every kind of projection the object should be quite stationary.*

## DEFINITION OF PERSPECTIVE.

*Linear Perspective* is the art of drawing upon a *plane* surface the resemblances of the outlines of objects when viewed from a particular station point.

*Aerial Perspective* is the art of giving due diminution to the strength of light, shade, and colour of objects, according to their distances and the quantity of light received by them, and to the medium through which they are seen.

The theory of perspective is founded upon the laws of vision; it is therefore necessary that the student should have a general idea of the construction of the human eye, and of the nature and cause of vision.

## CONSTRUCTION OF THE HUMAN EYE.

"The form of the eyeball is that of a sphere of about one inch in diameter, having the segment of a smaller sphere engrafted upon its front surface. The axes of the two eyeballs are parallel to each other, but do not correspond with the orbits, which are directed outwards."

"The globe of the eye is composed of *tunics*, and of refracting media called *humours*. There are three *tunics* and three *humours*."

First Tunic.—"The *sclerotic* and *cornea* form the external tunic of the eyeball, and give it its peculiar form. Four-fifths of the globe are invested by the sclerotic, and the remaining fifth by the cornea."

"The sclerotic is a dense fibrous membrane, thicker behind than in front. In front it presents a bevelled edge, which receives the cornea in the same way that a watch-glass is received by the groove in its case. Behind, it is continuous with, and pierced, by the optic nerve."

"The cornea is the transparent projecting layer that constitutes the front fifth of the globe of the eye. In its form it is circular, concavo-convex, and resembles a watch-glass."

Second Tunic.—"The second tunic of the eyeball is formed by the *choroid ciliary ligament* and *iris*. The choroid is a vascular membrane of a rich chocolate-brown colour upon its external surface, and of a deep black colour within. It is connected to the sclerotic, externally, by an extremely fine tissue; internally it is in simple contact with the third tunic of the eye, the *retina*. It is pierced behind for the passage of the optic nerve, and is connected in front with the iris, ciliary process, and with the line of junction of the cornea and sclerotic by a dense white structure called the ciliary ligament, which surrounds the circumference of the iris like a ring."

"The *ciliary ligament*, is the bond of union between the external and middle tunics of the eyeball, and serves to connect the cornea and sclerotic at their line of junction with the iris and external layer of the choroid. Thus this ligament is enabled to act upon the lens, and accommodate the focus of the eye to various distances."

"*The iris* is so named from its variety of colour in different individuals; it forms a partition between the front and back chambers of the eye, and is pierced somewhat to the nasal side of its centre by a circular opening called the pupil."

"The iris is composed of two layers, *a front, or muscular*, consisting of radiating fibres, which converge from the circumference towards the centre. having the power of dilating the pupil; and *circular*, which surrounds the pupil, and by their action produce contraction of its area. The back layer of the iris is of a deep purple tint, and is thence named *uvea*, from its resemblance in colour to a ripe grape."

"The *ciliary processes* consist of a number of triangular folds, formed apparently by the plaiting of the middle and internal layer of the choroid. They are about sixty in number, and may be divided into large and small, the latter being situated in the spaces between the former."

Third Tunic.—"The third tunic of the eye is the *retina*. It resembles a fine net; it lines the choroid, is prolonged forwards to the lens, and is connected behind with the optic nerve."

HUMOURS.—"*The aqueous humour* is situated in the back and front chambers of the eye; it is a weakly albuminous fluid, having a specific gravity very little greater than that of distilled water."

"The front chamber of the eye is the space intervening between the cornea in front and the iris and pupil behind. The back chamber is the narrow space between the iris and the lens. The two chambers are lined by a thin layer, *the secreting membrane of the aqueous humour.*"

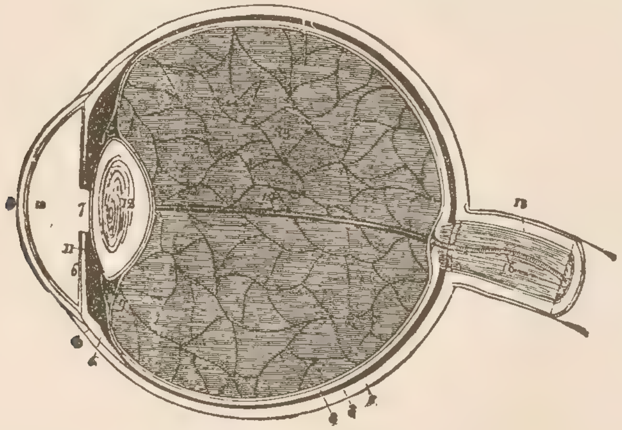


Fig. 2.

A longitudinal section of the globe of the eye. 1. The sclerotic, thicker behind than in front. 2. The cornea, received within the anterior margin of the sclerotic, and connected with it by means of a bevelled edge. 3. The choroid, connected anteriorly with (4) the ciliary ligament, and (5) the ciliary processes. 6. The iris. 7. The pupil. 8. The third layer of the eye, the retina, terminating anteriorly by an abrupt border at the commencement of the ciliary processes. 9. The canal of Petit, which encircles the lens (12); the thin layer in front of this canal is the zonula ciliaris, a prolongation of the vascular layer of the retina to the lens. 10. The anterior chamber of the eye, containing the aqueous humour; the lining membrane by which the humour is secreted is represented in the diagram. 11. The posterior chamber. 12. The lens, more convex behind than before, and enclosed in its proper capsule. 13. The vitreous humour enclosed in the hyaloid membrane, and in cells formed in its interior by that membrane. 14. A tubular sheath of the hyaloid membrane, which serves for the passage of the artery of the capsule of the lens. 15. The neurilemma of the optic nerve. 16. The arteria centralis retinæ, embedded in the centre of the optic nerve.

"*The vitreous humour* forms the principal bulk of the globe of the eye. It is an albuminous and highly transparent fluid, enclosed in a very delicate membrane."

"*The crystalline humour*, or *lens*, is situated immediately behind the pupil. It is more convex behind than in front, and is embedded in the front part of the vitreous humour, from which it is separated by the *hyaloid membrane.*"

"The sclerotic is a tunic of protection, and the cornea a medium for the transmission of light. The choroid supports the vessels destined for the nutrition of the eye, and by its black colour absorbs all loose and scattered rays that might confuse the image impressed upon the retina. The iris, by means of its power of expansion and contraction, regulates the quantity of light admitted through the pupil."

If the iris be thin, and the rays of light pass through its substance,

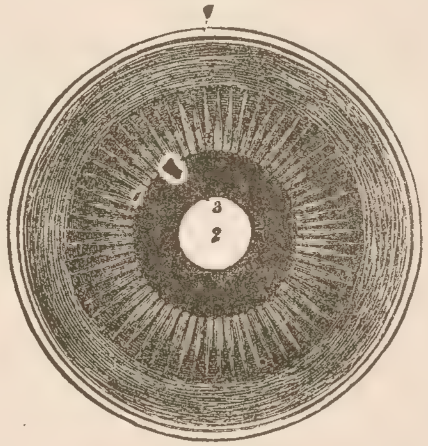


Fig. 3.

The anterior segment of a transverse section of the globe of the eye, seen from within. 1. The divided edge of the three tunics: sclerotic, choroid (the dark layer), and retina. 2. The pupil. 3. The iris, the surface presented to view in this section being the uvea. 4. The ciliary processes. 5. The scalloped anterior border of the retina.

they are immediately absorbed by the *uvea*, and if that layer be insufficient they are absorbed by the black pigment of the ciliary processes. If this pigment were absent, the rays of light would traverse the iris, and even the sclerotic, and so overwhelm the eye with light that sight would be destroyed.

"The transparent cornea and the humours of the eye have for their office the refraction of the rays of light in such proportion as to direct the image in the most favourable manner upon the retina. Where the refracting

medium is too great, as in over convexity of the cornea and lens, the image falls short of the retina and produces near-sightedness; and where it is too little the image is thrown beyond the membrane, which causes far-sightedness."

*⁎* *The above quotations are taken from " The Anatomists' Vade-mecum " by Erasmus Wilson, F.R.S.*

## THEORY OF THE NATURE AND CAUSE OF VISION.

Without light there would be darkness, and the object would be imperceptible.

" Every visible body emits or reflects inconceivably small particles of matter from every point of its surface, which issue from it continually in straight lines, and in all possible directions. These particles entering the eye fall upon the *retina*, and excite in our minds the idea of light, and as they differ in magnitude, they produce in us the ideas of different colours."

" A stream of these particles issuing from the surface of a visible body in one and the same direction is called a ray of light."

" That these particles which constitute light are exceedingly small may be proved by the following experiment. If a hole be made through a piece of paper with a needle, all the rays of light, which proceed at the same time from all the objects on one side of it, are capable of passing through at once without the least confusion; for any one of those objects may be as clearly seen through it, as if no rays passed through it from the other objects."

" It is evident that these particles proceed from every point of the surface of visible body in every direction, because wherever the spectator is placed with regard to the body, every point of that surface which is turned towards him is seen by him."

" That these particles proceed from the body in straight lines, we are assured, because just so many and no more will be intercepted in their passage to any place by an interposed object, as that object ought to intercept, supposing them to proceed in straight lines."—*Smith's Optics.*

For example, if we imagine ourselves in a room having the window shutters nearly closed, and the sun shining brightly immediately opposite the opening between the shutters, we should see the rays of light streaming into the room in *straight* lines, and illuminating a very small portion of the wall opposite the window, viz., that portion lying in a *direct* line with the sun and the aperture between the shutters.

A photographic camera is constructed upon the same principle as the human eye, and the image upon the retina is produced precisely in the same way as a photographic negative, by means of the refraction of the rays of light.

" In viewing an object, the rays of light which proceed from the different points of its surface are so refracted by the cornea and the crystalline humour as to converge and meet again in so many points upon the retina, and produce the image of the object upon it. This image, propagated by motion along the fibres of the optic nerve into the brain, is the cause of vision; for accordingly as the image is perfect or imperfect, the object is seen perfectly or imperfectly."—*Smith's Optics.*

## PRINCIPLES OF PERSPECTIVE.

It has been shown that objects are seen by means of rays of light which, in the practice of perspective, are represented by a series of straight lines. The rays of light *converge* to a point within the eye, and produce upon the retina an *inverted* image of the object from whence the rays proceed. If we imagine a transparent vertical plane placed somewhere between the object and the spectator's eye, it would intersect these rays, and if the points of intersection were joined, a perspective projection of the object would be thus obtained.

As a further illustration, let the spectator hold a sheet of glass vertically upon the table in front of a cube, or any other object, then, if he shut one eye, keeping it perfectly steady, and trace the visible boundary lines of the solid upon the glass, such tracing would be a correct perspective projection of the cube; for, if threads were made to pass through the

glass from the visible corners of the cube to the spectator's eye, they would represent the rays of light, and intersect at the corners of the tracing of the cube upon the glass.

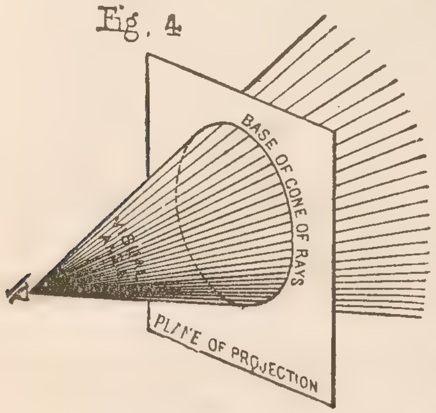
Perspective representations are obtained by using *one* eye. The apparent forms and dimensions of objects are much better ascertained with one than both eyes; for if both eyes be used, two visual impressions must necessarily be obtained.

If a piece of cardboard be held vertically, having its edge immediately opposite the left eye, *the right eye being closed*, the edge only will be seen; but if the left eye and piece of cardboard be kept in their original positions and the right eye be opened the plane of the cardboard as well as its edge will be perceptible.

In the practice of perspective the eye is always represented as a simple geometrical point.

As soon as the eye is opened, a quantity of light from the objects, and the whole of the surrounding space, streams into the pupil—each ray converges to it; consequently the mass of light takes the form of a cone. If the cone of rays be intersected by the vertical plane of projection, a circle will be produced, which is called "*base of cone of rays*," and the angle at the eye between the external rays of light is called *visual angle*. If we close one eye and keep the other quite stationary, it is obvious that our field of vision is very limited, and since we can draw in perspective only those objects of which we have perfect vision, it is absolutely necessary to determine how far we can obtain a distinct view on either side of the eye.

Fig. 4



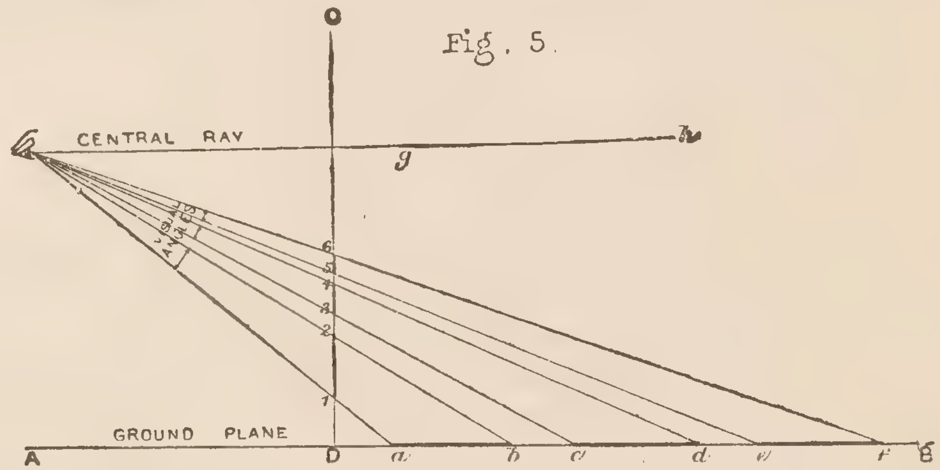

If we stand opposite to one of the walls of a room, and mark a point upon it as far as we can distinctly see on either side of us, the space between the points will represent the limit of our vision. If these points were joined to the eye by straight lines, the visual angle (the angle subtended at the eye) would be as nearly as possible one-sixth of a circle, or equal to 60 degrees.

Objects of equal magnitude and at the same distance from the eye appear of equal magnitude.

Objects of equal magnitude, placed behind each other at equal distances appear to approach each other, and diminish in magnitude as they recede from the eye.

Imagine three lines, **a b**, **c d**, **e f**, FIG. 5, of equal lengths placed equally distant from each other, and lying upon the ground-plane **A B**.

Fig. 5.




The vertical plane of projection is shown between the objects and the eye at **C D**. *N.B.*—The illustration is a *profile* of the planes, lines, &c.

In order to find the perspective representation, we have simply to draw the rays of light from the extremities **a b**, **c d**, **e f** to the eye, and where they intersect the plane of projection we shall have the images of the points from whence they were drawn; **1 2**, **3 4**, **5 6**, are the images of the given lines.

If FIG. 5 be carefully examined, the student will perceive that the images of the lines and the spaces between them diminish in proportion to the angle at the eye made by the rays of light drawn from their extremities; for example, the angle at the eye, made by the rays of light from **a** and **b**, is larger than that made by the rays from **c** and **d**; because in the former

case the rays are more obliquely situated with regard to the eye and plane of projection than in the latter; and for the same reason the image of **a b** is larger than that of **c d**.

Again, the line **c d** being nearer to the eye and plane of projection than the line **e f**, its visual angle is larger than that of **e f**, consequently its image is proportionately larger.

Lastly, imagine a line **g h** to be placed horizontally having its extremity **g**, immediately opposite the spectator's eye; it is manifest there would be no visual angle, because the rays of light coincide with the central ray, therefore, the line **g h** would appear as a simple point.

From what has been said, it is clear that the apparent size of an object is determined by the magnitude of the visual angle.

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## DEFINITIONS.

The planes most frequently used in perspective are,

I. Picture-Plane, or Plane of Projection, is, as previously described, a transparent* vertical plane supposed to exist somewhere between the spectator's eye and the object to be represented. It is the plane through which the object is viewed, and upon which its image is described. The sheet of paper upon which the student makes his perspective drawing is the picture-plane.

II. Ground Plane is the plane on which the objects rest. It is at right angles to, and supports the picture-plane.

III. Directing Plane is an imaginary plane drawn through the spectator's eye parallel to the picture-plane.

IV. Vanishing Plane is an imaginary plane drawn through the spectator's eye to meet the picture-plane parallel to *any* original plane.

V. Perpendicular Plane is an imaginary plane drawn through the spectator's eye perpendicular to the four preceding planes.

The student must now refer to Fig. 6, and endeavour to understand the positions and intersections of the planes, for his ultimate success will depend in a great measure upon an accurate knowledge of these elementary planes.

**A B C D** represents the picture-plane standing vertically upon the ground plane **C D E F**; the two planes intersect each other in the line **A B**.

Point $\odot$ represents the position of the spectator's eye in front of the plane.

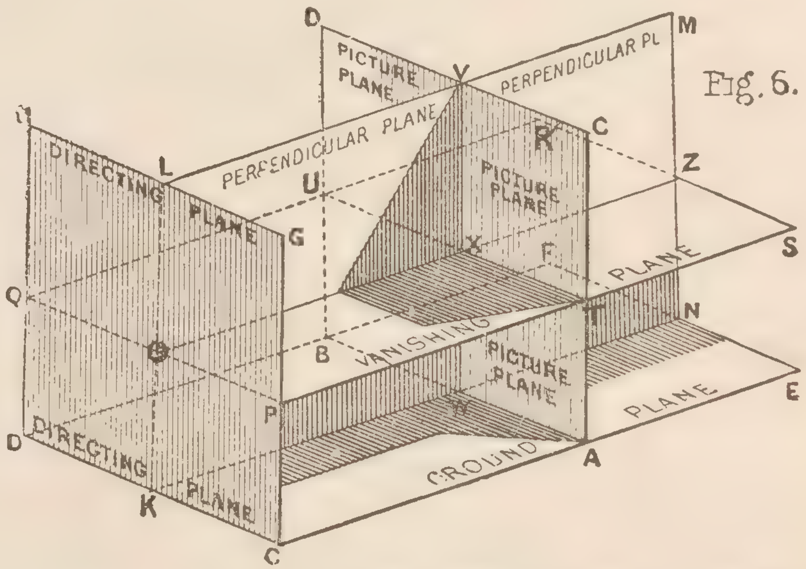



Fig. 6.

**C D G H** is the directing plane drawn through $\odot$ parallel to the picture-plane; it intersects the ground plane in the line **C D**.

*The directing and picture planes are parallel, therefore, their intersections with the ground-plane are parallel.*

**P Q R S** is a vanishing plane; it is the vanishing plane of the ground, because it is drawn through $\odot$ parallel to that plane. It intersects the directing plane in the line **Q P**, and the picture-plane in the line **T U**.

**K L M N** is the perpendicular plane; it is perpendicular to all the other planes. It intersects the *ground-plane* in the line **K N**, the *picture plane* in the line **V W**, the *directing-plane* in the line **L K** and the *vanishing plane* in the line $\odot$ **Z**.

* The Picture-Plane need not always be vertical; it may be in any position whatever, and will depend upon the direction in which the spectator views the object, for the picture-plane must always cut the axis of vision at *right angles*.

VI. Ground Line, or Picture Line, is the intersection of the ground-plane with the picture-plane, as **A B**, FIG. 6.

VII. Vanishing Line is the intersection with the picture-plane of an imaginary plane passing through the eye parallel to any original plane.

The line **T U**, FIG. 6, is the vanishing line of the ground-plane, because the line **T U** is the intersection with the picture-plane made by an imaginary plane passing through the eye parallel to the ground-plane.

Again, the line **V W** is the vanishing line of *any* vertical plane perpendicular to the picture-plane, because a similar plane drawn through the eye cuts the picture-plane in this line.

VIII. Directing line is the intersection of any plane with the directing-plane, as the lines **C D**, **P Q**, FIG. 6.

*The principal directing line is* **P Q**. It is always required in the practice of perspective; the student is therefore recommended to give it special attention.

*The directing and picture planes are parallel, therefore, if a third plane be made to cut them, their common intersections would be parallel lines;* for example, *the vanishing plane cuts the directing and picture planes respectively* in the parallel lines **P Q**, **T U**.

IX. Horizontal Line is the line in the extreme distance which bounds our view; it is the line in which the sky and earth appear to meet; it is the level of the spectator's eye, and is found upon the picture-plane by the intersection of an imaginary plane passing through the eye parallel to the ground-plane.

The line **T U**, FIG. 6, represents the horizontal line.

X. Intersecting Line is the intersection with the picture-plane of *any* plane situated behind it; for example, the line **A B**, FIG. 6, is the intersecting line of the ground-plane, because the ground plane meets the picture-plane in this line.

N.B.—The intersecting line of a vertical plane is sometimes called "Line of Heights," "Measuring Line," and Picture Line.

XI. Original object is any object behind the picture-plane whose perspective representation is required.

XII. Original Plane is any plane of which an original object is composed.

XIII. Original Line is any line which bounds the original object.

XIV. Visual Ray is an imaginary ray of light represented by a straight line drawn from any point in an original object to the spectator's eye.

XV. Visual Angle is the angle at the spectator's eye formed by the external rays of the cone of rays. (See FIGS. 4, 5, 7.)

XVI. Station Point is the position selected by the spectator in front of the picture-plane, from which he views the object.

By referring to FIG. 6 it will be seen that the station point (spectator's eye) lies in the intersection of the *directing*, *vanishing*, and *perpendicular planes*.

*The station point is the apex of the imaginary cone of rays.* (See FIG. 7.)

N.B.—Throughout this work the station point is called EYE.

XVII. Principal Visual Ray, is the axis of the imaginary cone of rays; it is the shortest ray of light which can be drawn between the spectator's eye and the picture-plane.

The principal visual ray bisects the visual angle, and is *perpendicular* to the plane of the picture. (See FIGS. 7, 8.) It is the intersection of the *vanishing* and *perpendicular planes*, and is shown by the line ⊙ **X**, FIG. 6.

The principal visual ray is sometimes called "Line of Direction," because it indicates the way in which the spectator views the object. It is also called "axis of vision."

XVIII. Centre of vision is the point on the picture-plane immediately opposite the spectator's eye. It is the point of intersection of the principal visual ray with the picture-plane. It is always the centre of the base of the cone of rays.

It is represented at **C V**, FIG. 7, also at **X**, FIG. 6.

XIX. Distance Points are points set off on the horizontal line, on

either side of the centre of vision, equal to the distance of the eye in front of the picture-plane.

If we imagine the principal visual ray to revolve in the vanishing plane upon the centre of vision as centre, the station point, or Eye, would describe a semicircle, and come into contact with the picture-plane, upon the horizontal line, at the points **D 1** and **D 2**, Fig. 8.

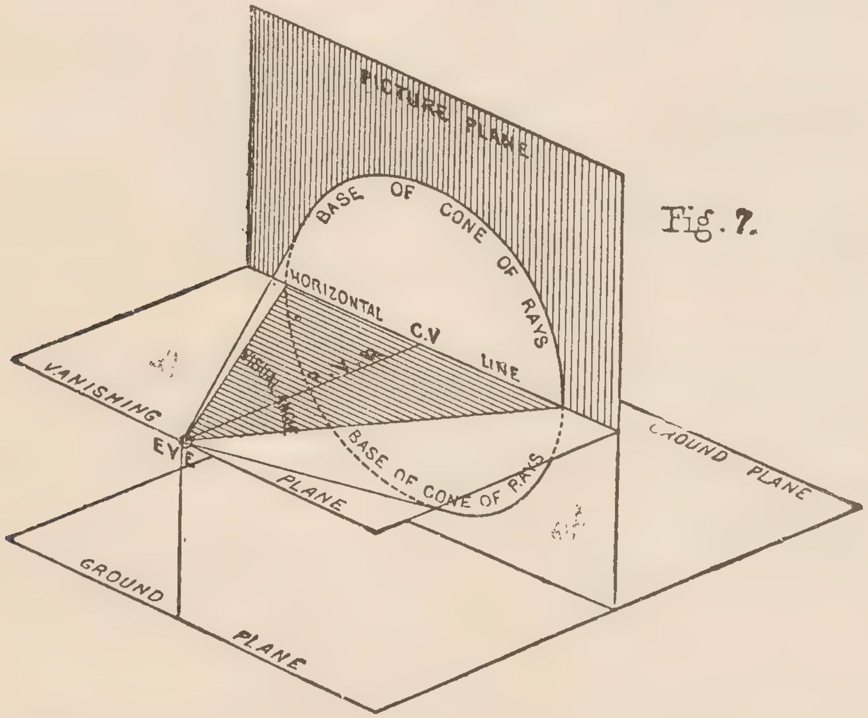



Fig. 7.

XX. Vanishing Point is a point to which parallel lines appear to converge.

If a line be drawn from the eye parallel to any original line, its point of intersection with the picture-plane is the vanishing point of such original line. As an illustration, the student is referred to Fig. 7, the principal visual ray is drawn from the eye perpendicular to and intersecting the picture-plane at **C V.** The centre of vision is therefore the vanishing point of any or all original lines parallel to the principal visual ray.

N.B.—The vanishing point of a line inclined to the ground plane is called "Accidental Vanishing Point."

XXI. Vanishing Parallel is a line drawn from the eye, parallel to any original line to intersect the picture-plane; the point of intersection is the vanishing point of the original line.

XXII. Measuring point is a point by which perspective measurements are obtained. The distance between a vanishing point and its measuring

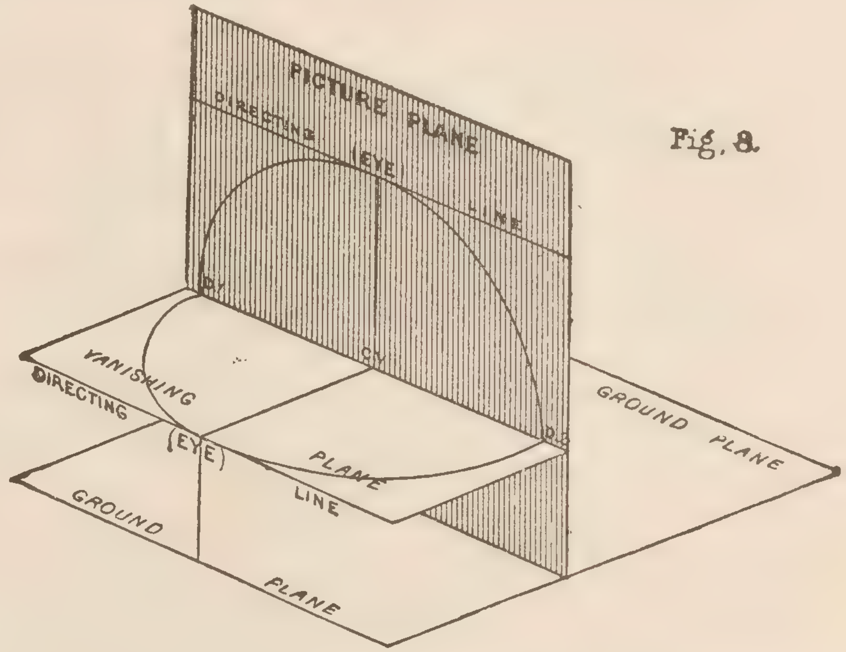



Fig. 8.

point must always equal the distance of the eye from the vanishing point.

It has been shown (Definition XX.) that the **C V** is the vanishing point of lines parallel to the principal visual ray. Now, if it were required to find the measuring point of the vanishing point **C V**, we have merely to set off the distance of the eye from **C V**, and **D 1** or **D 2** will become the measuring point, Fig. 8.

XXIII. Centre of vanishing line is obtained by drawing from the eye a perpendicular to the vanishing line.

XXIV. Base of cone of rays is a section of the cone of rays made by the picture-plane. It is the limit of our field of vision.

N.B.—The nearer the cone is cut to the apex, the more limited our field of vision becomes.

## OBSERVATIONS.

I. The *shortest* line which can be drawn between any point and a plane is a *perpendicular* to that plane.

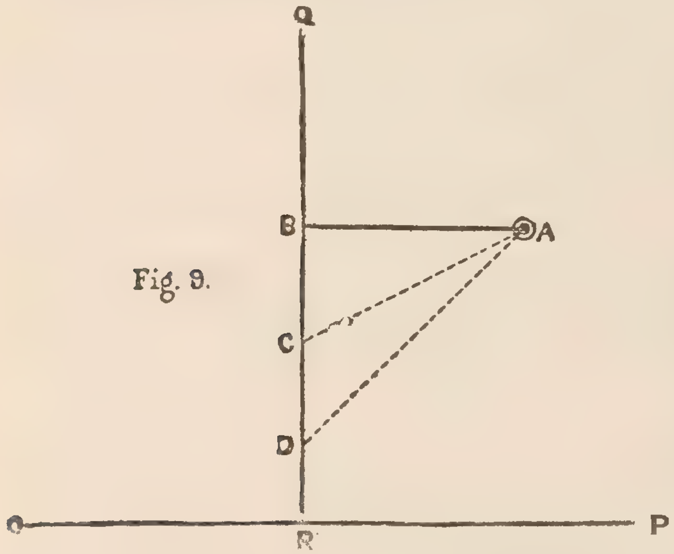


Fig. 9.

Let **O P**, **Q R**, Fig. 9, be the profile of two planes at right angles to each other. Point **A** is the given point.

Draw three straight lines from **A** to meet the profile of the vertical plane at points **B C D**. The line **A B** to be perpendicular to the plane; the lines **A C**, **A D**, may be drawn at pleasure. It will be found that the perpendicular **A B** is shorter than either of the other lines, and consequently must be the actual distance between the given point **A** and the plane **Q R**.

II. *A line may be parallel, perpendicular, or inclined to the picture-plane.*

A line may be parallel to the picture-plane, and at the same time make any angle with the ground. (See Fig. 10.)

The ground-plane is given, and the picture-plane supposed to stand vertically upon the line **O P**, as shown by the dotted lines.

Four lines, **A B**, **A C**, **A D**, **A E**, are given in a vertical plane parallel to and behind the picture-plane. Because these four lines lie in a *plane parallel* to the *picture*, it is obvious they must likewise be parallel to the picture-plane, for if a *plan* of the lines be obtained, they would appear in one line parallel to the ground-line.

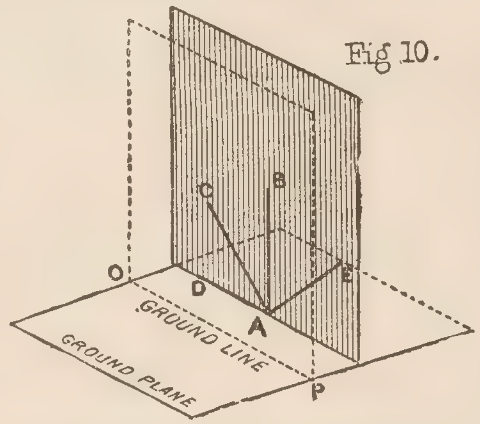


Fig 10.

The line **A D** is *parallel* to the *picture* and *ground-planes.* The line **A B** is *parallel* to the *picture-plane,* but *perpendicular* to the *ground-plane.* The lines **A C**, **A E** are *parallel* to the *picture-plane,* but make certain *angles* with the *ground-plane.*

III. *A plane may be parallel, perpendicular, or inclined to the picture-plane.*

A plane *parallel* to the *picture-plane* can have but one direction, viz., *vertical,* as **A B C D**, Fig. 11.

We may now imagine the vertical plane **A B C D** revolved upon **A B** as an axis so as to occupy the position **A B E F**, or **A B G H**, it would then be called a *direct ascending-plane;* or if the plane **A B C D** were revolved towards the picture-plane, instead of away from it, it would be a *direct descending-plane.* The plane in either case is called *direct,* because it has its upper and lower edges *parallel* to the *picture-plane.*

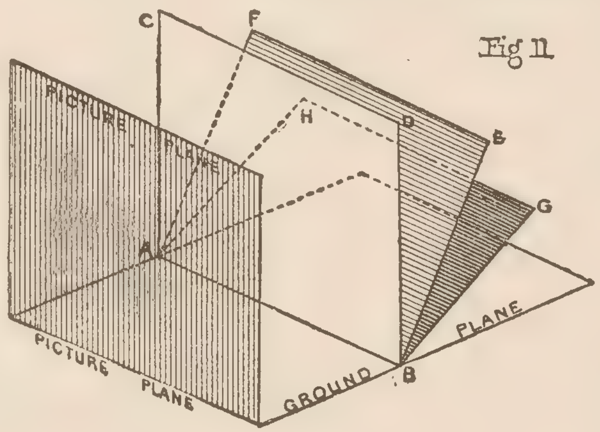
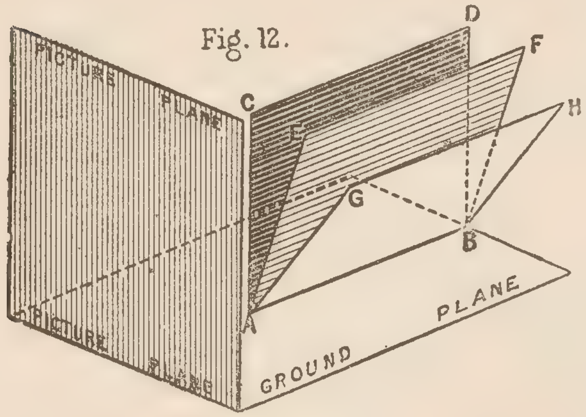


Fig 11.

*A plane perpendicular to the picture-plane may be parallel, perpendicular, or inclined to the ground at any angle whatever.*

The plane **A B C D**, Fig. 12, is *perpendicular* to the *picture-plane,* and it is likewise *perpendicular* to the *ground-plane.*

*The vanishing-plane*, Fig. 6, will serve as an illustration of a plane *perpendicular* to the *picture*, but *parallel* to the *ground-plane*.



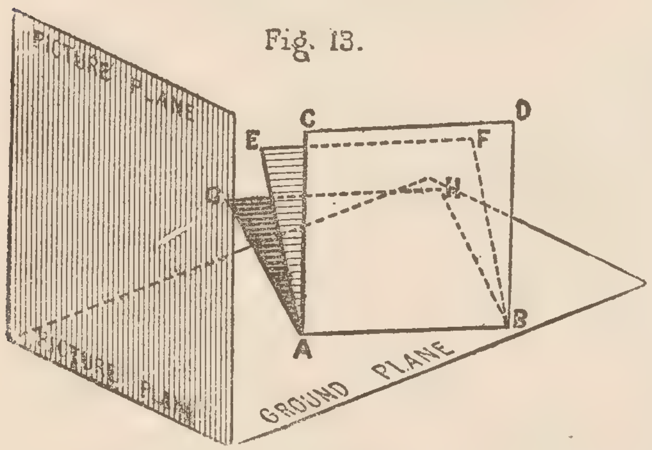

The planes, **A B E F A B G H**, Fig. 12, are *perpendicular* to the *picture-plane*, but *inclined* to the *ground-plane*.

---

*A plane inclined to the picture-plane may be perpendicular or inclined to the ground-plane at any angle whatever.*

**A B C D**, Fig. 13, is a plane *inclined* to the *picture-plane*, but *perpendicular* to the *ground-plane*.

If we imagine the plane **A B C D** revolved upon **A B** as an axis to occupy the position **A B E F**, or **A B G H**, it would then be inclined to both *ground* and *picture-planes*.




The planes **A B E F**, **A B G H** are called *oblique ascending-planes*. If the plane **A B C D** were revolved towards the picture-plane, instead of from it, it would then be called an *oblique descending-plane*.

*N.B.*—Planes *parallel* to the *picture-plane* have their intersections with the *ground-plane* parallel to the *ground-line*. (See **D C** Fig. 6.)

Planes *perpendicular* to the *picture-plane* have their intersections with the ground-plane *perpendicular* to the *ground-line*. (See **K N**, Fig. 6.)

Planes *inclined* to the *picture-plane* have their intersections with the ground-plane *inclined at an equal angle* with the *ground-line*; for example the plane **A B C D**, Fig. 13, is inclined to the picture at a certain angle, and its intersection with the ground-plane is a line inclined to the ground-line at an equal angle.

IV. The picture-plane is supposed to be of indefinite extent; it is necessary to imagine it produced *below* the ground-plane. (See Fig. 14.)

---

V. The horizontal line requires consideration, because at different heights of the eye the object appears different in form, &c.; for example, if the eye be above the object, its upper surface will be visible, but if the top of the object be ever so little above the eye, it will be invisible. If the horizon be too high or too low, the object would have a distorted appearance.

A view taken from an eminence, a cliff for instance, would require a high horizontal-line; if a view be taken from level ground a low horizon would be appropriate.

Objects appear more natural from a level of about five feet above the ground-plane, because this is about the average height of a man's eye, and the position from which we are accustomed to view them.

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VI. The distance of the eye in front of the picture-plane, in every case, should be greater than the height of the eye above the ground-plane and never less than the diagonal of the picture, or the spectator would be unable to see the whole group of objects.

---

VII. The object may touch the picture-plane, or it may be any distance behind the picture-plane, so long as it can be distinctly seen from the station point.

---

VIII. If the cone of rays, Fig. 7, be cut by the vanishing-plane passing through its axis, the form of section is a triangle, and since the cone of rays has an angle of 60° at its apex, the true form of section must be an *equilateral triangle*.

This triangle is constructed by making an angle of 30 *degrees* on either side of the eye, with the *principal visual ray,* or 60 *degrees* on either side of the eye with the *directing-line*; for if a semicircle be described from the eye as centre, it will contain 180 degrees, and if 60 degrees be cut off on

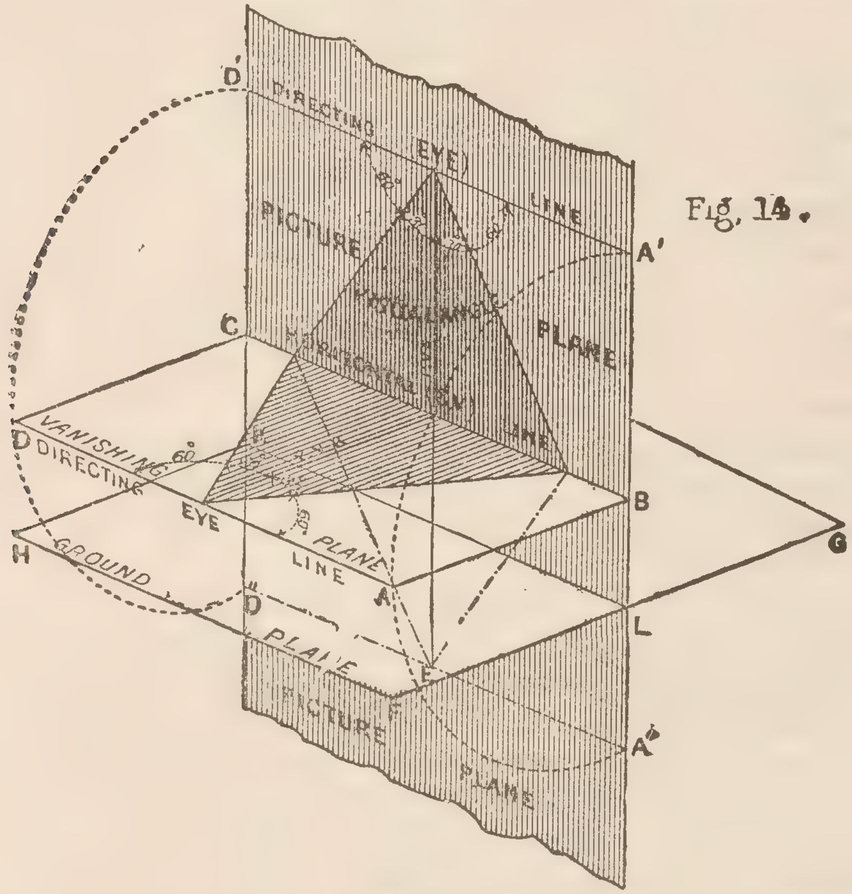


Fig. 14.

either side of the eye with the directing-line, an arc of 60 degrees will remain in the centre which is bisected by the principal visual ray, FIG. 14.

---

IX. In the horizontal line are found the *centre of vision, distance points,* and *vanishing* and *measuring points* of all horizontal lines inclined to the picture-plane.

X. In order to make a perspective drawing of an object, we must know the distance of the eye from the picture-plane, the height of the eye above the ground-plane, the actual dimensions of such object, and its position with regard to the picture-plane.

XI. All lines and plane surfaces in contact with the picture appear of their actual forms and dimensions.

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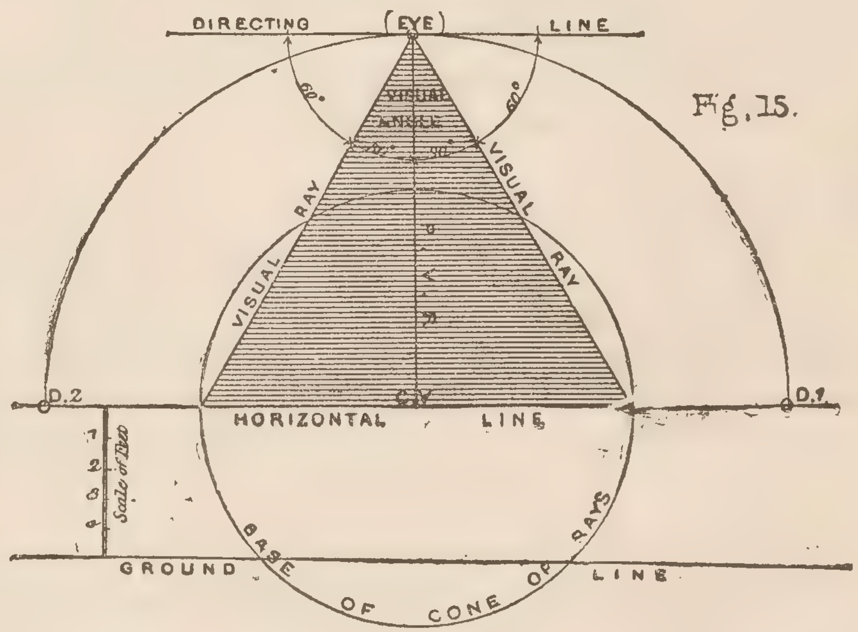



Fig. 15.

## METHOD OF PREPARING THE PICTURE-PLANE.

Let the spectator's eye (station point) be 12′ in front of the picture-plane and 5′ above the ground-plane. (Scale ¼″ to 1′.)

In the practice of perspective the picture-plane is represented by the paper upon which the drawing is made, and although the eye, and the lines and planes drawn from it, are actually in front of the picture-plane, they cannot be thus represented, for it is very evident that the station point cannot be indicated by a point at 12′ in front of the drawing paper.

The first line to be determined upon the picture-plane is the *horizontal*

*line.* The student should ask himself what is the horizontal line. It is the intersection with the picture-plane of an imaginary horizontal plane drawn through the eye. **A B C D**, FIG. 14, represents this plane in its actual position; the spectator does not really see the plane, he simply sees its edge **A D**, which appears to him to coincide with the horizontal line, because the plane is level with his eye.

The horizontal line is usually placed at about a third from the lower or upper edges of the drawing paper (See FIG. 15). It should be drawn across the paper of indefinite length.

The height of the eye above the ground-plane is shown upon the picture-plane between the horizontal line **C B**, and the ground-line **K L**, FIG. 14.

The height of the eye upon the drawing paper is to be **5′**; therefore *below* the *horizontal line* set off five equal parts, each part being equal to ¼″, and representing **1′**; then through the fifth point of the scale draw the ground-line of indefinite length, parallel to the horizontal line, FIG. 15.

*N.B.—The ground-plane is represented of indefinite extent between the horizontal line and the ground-line.* The ground-line represents the near edge of the ground-plane in contact with and behind the picture-plane, and the horizontal-line represents where the ground-plane, in receding from the eye appears to vanish.

The centre of vision is shown at **C V**, FIGS. 7, 8, 14, 15.

We may imagine our eye opposite any point on the horizontal line; it is, however, usually placed at or near the centre.

Fix **C V**, FIG. 15, near the centre of the horizontal line.

The distance of the eye in front of the picture-plane must now be considered. It is shown in its actual position by the length of the principal visual ray projecting at right angles to, and in front of the picture-plane, FIG. 14.

It is simply impossible to represent the principal visual ray at right angles to the surface of the drawing paper; therefore we must imagine it to revolve in the vanishing-plane upon the horizontal line **C B** as an axis until it coincides with the picture-plane, FIG. 14.

The *vanishing-plane* may be made to coincide with the *picture-plane*, either *above* or *below* the *horizontal line.* In revolving upwards, the corners **A** and **D** of the vanishing plane will travel in the dotted lines, arcs **A A′**, **D D′**, and meet the picture-plane at points **A′** and **D′**; but if the vanishing plane be revolved downwards, its corners **A** and **D** will describe the arcs **A A″** and **D D″**, meeting the picture-plane at **A″** and **D″**, FIG. 14.

The student must carefully observe that whether revolved *above* or *below*, the *principal visual ray* is *perpendicular* to the *horizontal line*, during every portion of its revolution, and when in contact with the picture-plane it becomes a *vertical line.*

N.B.—The principal visual ray is usually placed above the horizontal line; but if we wish to economise the space or use a larger scale, the horizontal line should be placed at about a third from the upper edge of the drawing paper, and the principal visual ray *below* it, at *right angles.*

In the illustration, FIG. 15, the horizontal line is drawn at about a third from the lower edge of the paper, and the principal visual ray is drawn vertically above it, equal in length to twelve parts from the scale. The upper extremity of the principal visual ray is the position of the spectator's eye (station point).

Draw the *directing line* through the eye parallel to the ground and horizontal lines, because it occupies this position when the vanishing-plane is brought into the surface of the picture-plane. (See **A′ D′**, FIG. 14.)

In order to determine the visual angle, FIG. 15, take the eye as centre with any radius and describe a semicircle; then divide the semicircle into three equal parts by setting off the radius, from the directing line, thrice upon its circumference; each part contains 60 degrees, and the centre part is sub-divided into two equal portions, each containing 30 degrees.

Join the two points upon the semicircle to the eye, and produce the lines to meet the horizontal line. These lines (visual rays) form at the eye the visual angle, and, being produced to meet the horizon, determine the diameter of the base of cone of rays.

The triangle formed by the visual rays and the diameter of the base of the cone of rays is shaded, that the student may more clearly under-

stand it as being a section of the cone of rays made by the vanishing-plane passing through the eye and containing the principal visual ray. It is very clearly illustrated in its actual position, FIG. 7.

The base of the cone of rays is determined by drawing a circle from **C V** as centre through the extremities of the visual rays upon the picture-plane, FIG. 15.

The distance points have now to be found, which will complete the preparation of the picture-plane.

Imagine the principal visual ray to revolve upon **C V** as a centre; its extremity (the eye) will describe a semicircle in the vanishing plane, and meet the picture plane on the horizontal line at points **D 1**, **D 2**. (See FIG. 8.)

The actual position of the semicircle is in front of the picture-plane, but we have to imagine it folded into the plane of the picture upon the horizontal line as an axis.

Take **C V** as centre, FIG. 15, with the length of the principal visual ray as radius, and describe the semicircle in the plane of the picture, giving points **D 1**, **D 2**, on the horizontal line.

*N.B.—It is not absolutely necessary to find the base of the cone of rays every time we wish to make a perspective drawing, for we need only think of particular rays of light—those which proceed from the angles—or principal points of the surfaces of the object to be delineated.*

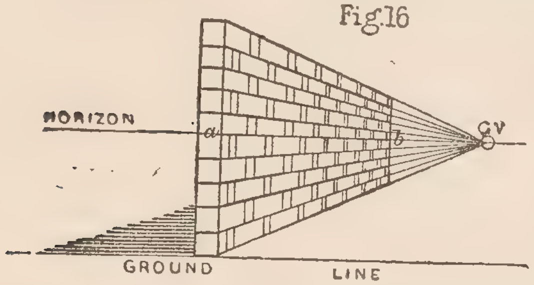
# RULES.

A. *Lines appear to converge to points.*

If a line be placed with its end immediately opposite the spectator's eye it will appear a point only. (See **g h**, Fig. 5.)

*Parallel lines appear to converge to the same point.*

Fig.16




The spectator is supposed to view the vertical wall, Fig. 16, in the direction of its length, the *horizontal mortar joints* although parallel amongst themselves, appear to converge to **C V.** The mortar joints *above* the horizon appear to tend downwards, those *below* it appear to tend upwards, while the joint **a b**, being on a level with the eye, appears to coincide with the horizon.

N.B.—There is one exception to Rule A, viz., in the case of lines *parallel* to the *picture-plane.* (See lines **a b**, **c d**, Figs. 17, 18.)

---

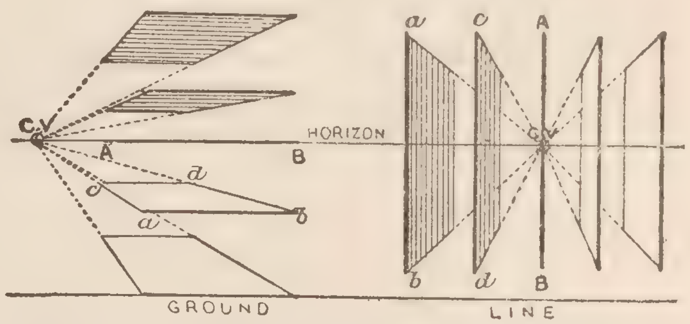
B. *Planes appear to converge to lines.*

If a plane be placed horizontally or vertically, having its edge immediately opposite the spectator's eye, it will appear a straight line. (See **A B**, Figs. 17, 18.)

*Parallel planes appear to converge to the same line.*

Figs. 17, 18, are the perspective representations of five parallel planes differently situated with regard to the picture-plane. The centre plane in both cases (**A B**) appears a straight line, because it is immediately opposite the spectator's eye. The parallel planes on either side of the centre plane appear to approach each other as they recede from the eye; and if they were sufficiently produced they would appear to vanish in **A B**.

Fig. 17 Fig. 18.




N.B.—There is an exception to Rule B., viz., when the parallel planes are parallel to the picture-plane. (See Fig. 19.)

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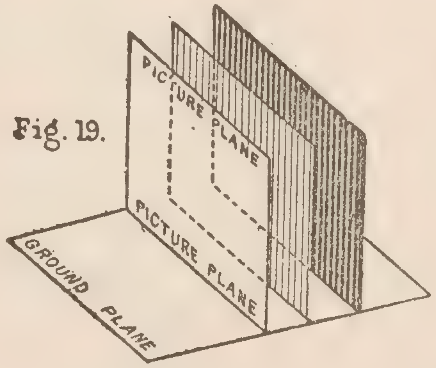
Fig. 19.




C. *Every line which vanishes must be measured by its own measuring point.*

We will imagine the line **A B**, Fig. 20, to lie upon the ground-plane on the spectator's left, *perpendicular* to the picture-plane, and its near extremity (**A**) touching the plane of the picture.

We must first obtain the vanishing point of **A B** by drawing a line from the eye parallel to **A B**, to meet the picture-plane.

*The principal visual ray* becomes the *vanishing parallel* of the line

**A B**, and the *centre of vision* is its *vanishing point.* (See Definitions XX., XXI.)

Since **C V** is the vanishing point of the line **A B**, it is obvious that the perspective representation must lie upon the line joining **A** to **C V**.

Join **A** to **C V**, then draw a line, representing a ray of light, from **B** to the eye, which intersects the picture-plane, on the former line, at point **b**.

**A b** is the perspective representation of **A B**.

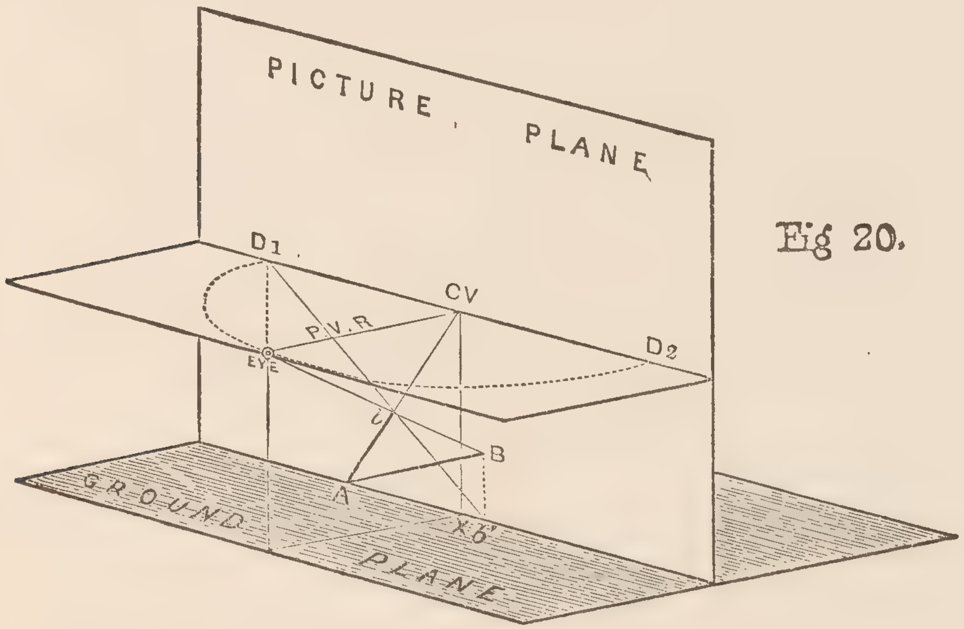



Fig 20.

N.B.—Observe that the line which represents the ray of light drawn from **B** to the eye determines the perspective length, hence the eye is the *measuring point* of **A B**. Point **D 1** represents the eye on the picture-plane, and is therefore the *measuring point* of the line **A B**.

In practice, we set off **A b′** on the ground-line equal to the actual length of **A B**, then join **b′** to **D 1**, which gives **A b**, the perspective length of **A B**, as previously obtained.

As a further proof that **A b** is correctly found in perspective, we will join **B b′** so as to form an isosceles triangle **A b′ B**, lying on the ground-plane. One of its equal sides **A b′** is shown on the picture-plane of its actual length, the other equal side is represented by the line **A B** and **b′ b** is the representation of the base **b′ B**. Again, if we draw the vanishing parallel of **b′ B** (*dotted line from* **EYE** *to* **D 1**) we shall have **D 1** as the vanishing point of **b′ b**, therefore **A b′ b** is the representation of the triangle **A b′ B**.

---

D. *Lines perpendicular to the picture-plane vanish to the centre of vision, and are measured by the distance points.* (See Fig. 20.)

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E. *Lines parallel to the picture-plane do not vanish, but are measured by the centre of vision.*

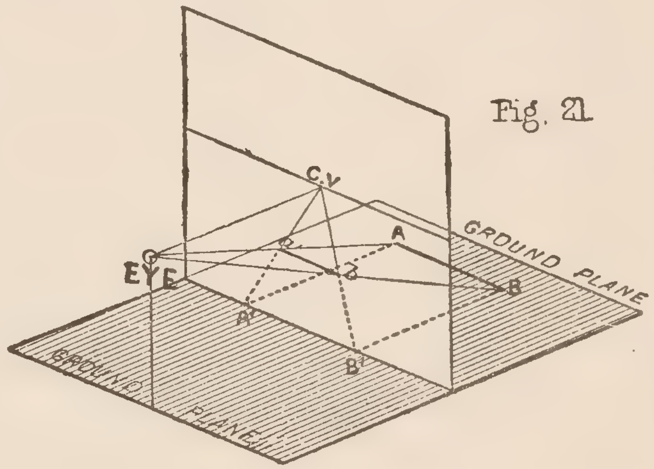


Fig. 21.

Let **A B**, Fig. 21, be a line lying upon the ground-plane, parallel to and behind the picture-plane. Required its perspective representation.

It has been previously shown (Fig. 9) that the actual distance between any original point and a plane is measured upon a line drawn from the point *perpendicular* to the plane.

Determine the actual distances of the extremities of the given line beyond the picture-plane by drawing the dotted lines **B B′**, **A A′**, perpendicular to the picture-line.

The perspective representations of the dotted lines **B B′**, **A A′**, must converge to **C V**, because these lines are perpendicular to the picture-plane. (Refer to Fig. 20, Rule D.)

Join **B′**, **A′** to **C V**, and draw lines to represent rays of light from **A B** to the eye, intersecting the picture-plane at **a** and **b**.

Lastly, join **a b**, and the line thus found will not vanish but keep parallel to the picture-line, which proves the above rule.

PLATE A.

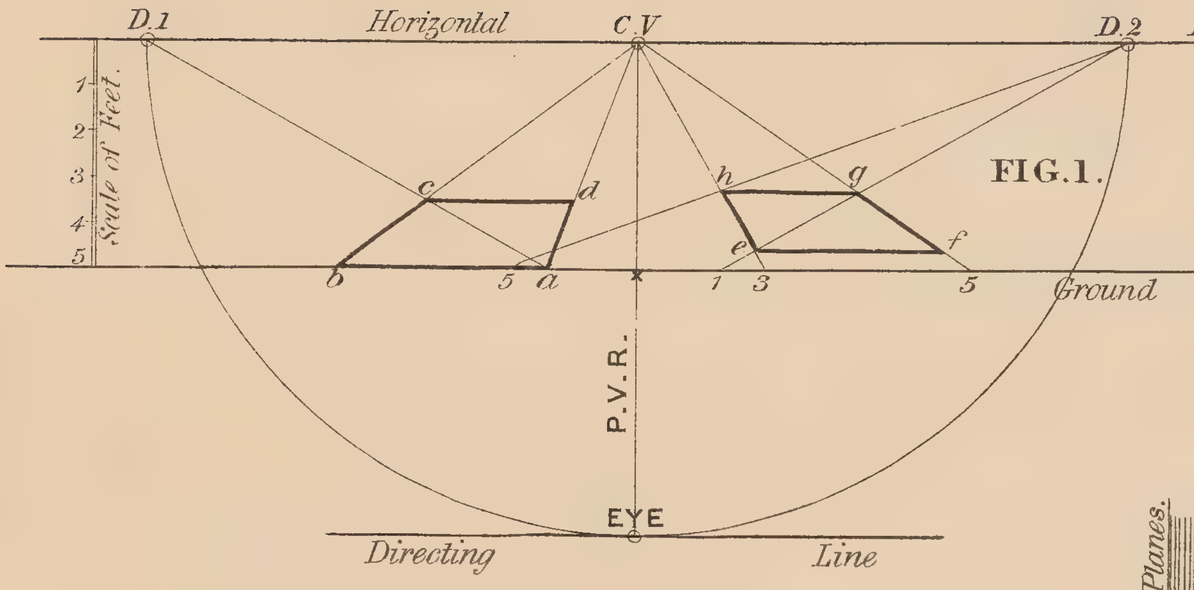


FIG.1.

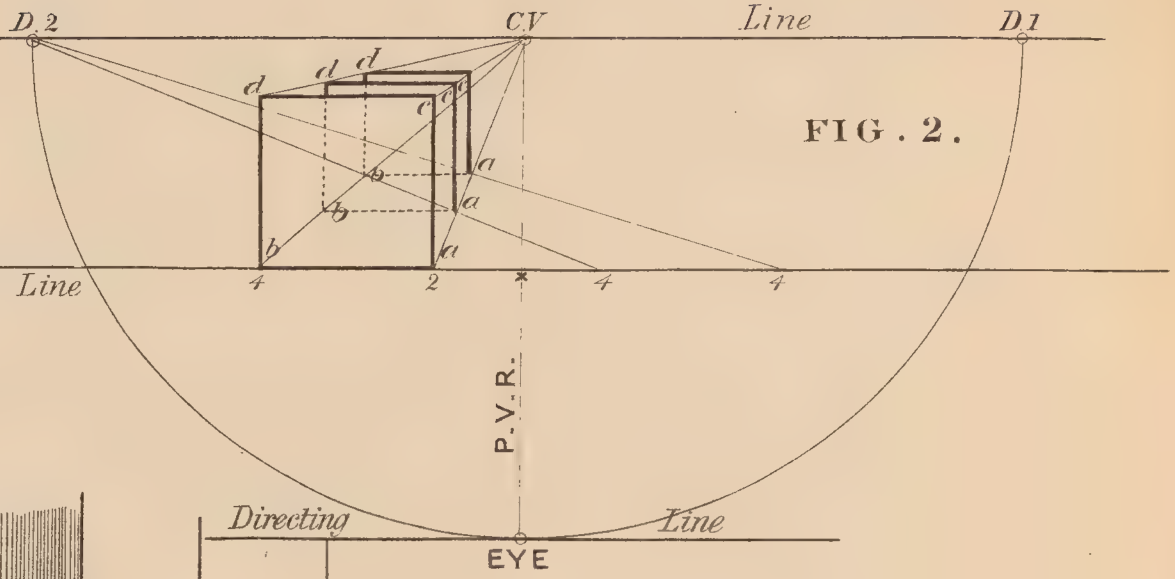


FIG. 2.

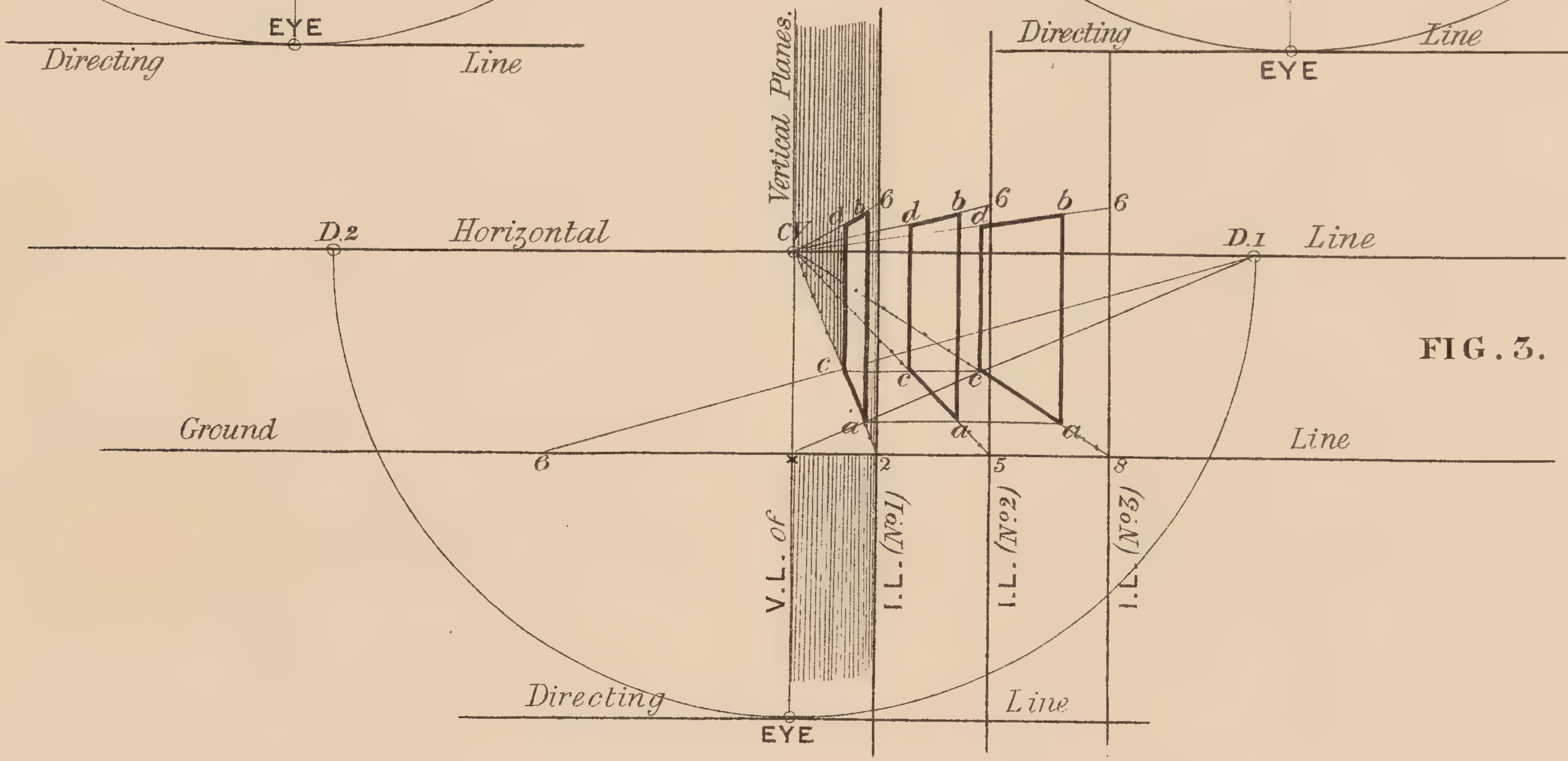


FIG. 3.

H.J. Dennis.

# PARALLEL PERSPECTIVE.

## PLATE A.

Distance of eye in front of the picture-plane, **11′**.
Height of eye above the ground-plane, **5′**.

FIG. 1.

I.—Find a point **a** lying upon the ground-plane, coincident with the ground-line at **2′** on the spectator's left.

II.—From point **a** draw a line, **a b**, **5′** long, towards left, lying upon the ground-line. Complete the perspective representation of a square lying upon the ground-plane, supposing that **a b** is its nearest edge.

III.—Find a point **e** lying upon the ground-plane, **1′** from the picture-plane, and **3′** on the spectator's right.

IV.—From point **e** draw a line **e h**, **5′** long, lying upon the ground-plane and receding from the picture-plane at right angles. The line **e h** is one edge of a square which lies upon the ground-plane *wholly* on the spectator's right.

---

*The student must prepare his paper for the perspective drawing as previously described for* FIG. 15.

The position of a point upon the ground-line immediately opposite the spectator's feet must be found, because all distances on right or left of the spectator and lying upon the ground-plane, are set off from it. In FIG. 6, it is represented by point **W**, the intersection of the perpendicular plane with the ground-line. It is immediately below the centre of vision, therefore let fall a perpendicular from **C V** to meet the ground-line at point **X**.

N.B.—Throughout this work the above-mentioned point is lettered **X**.

Upon the ground-line, towards left, set off **2′** from **X**, giving the required point **a**.

Again, from point **a** set off **5′** towards left, which determines **b**. The line **a b** is its actual length because it is on the picture-plane.

Since the line **a b** is on the ground-line, the adjacent edges of the square must necessarily be perpendicular to it, and consequently perpendicular to the picture-plane.

It has been shown (Rule **D**) that lines perpendicular to the picture plane vanish to **C V**, and are measured by **D 1** or **D 2**.

Join points **a** and **b** to **C V**, then measure one of the lines thus formed by **D 1** or **D 2** (say that drawn from **b**).

Upon the ground-line, set off **5′** from **b**, towards right, coinciding with point **a**, and join it to **D 1**, which gives one of the back corners of the square, point **c**, on the line **b C V**.

Now complete the square by drawing **c d** parallel to **a b**.

---

### SOLUTION OF SECOND POSITION (FIG. 1).

Upon the ground-line, towards right, set off **3′** from **X**, and join point **3** to **C V**.

Set off **1′** from point **3**, towards left, and join point **1** to **D 2**. The intersection of these lines is the required point **e**.

The line **e h** is to recede from the picture-plane at right angles, con-

sequently its perspective representation will lie upon the line joining **3** to **C V** (*See* Rule **D**).

Bring forward **e** by **D 2** to meet the ground-line at point **1**, then set off 5′ towards left, and if this point be joined to **D 2** the straight line will intersect **3 C V** at point **h**.

Through points **e** and **h** draw lines parallel to the picture-plane, the front and back edges of the required square will be found somewhere upon these lines, and according to Rule **E**, they will be measured by **C V.**

Since point **e** is beyond the picture-plane, the front edge of the square cannot appear its actual length (*See* Obs. XI.). It is necessary to bring forward **e** by its measuring point (**C V**) to coincide with the plane of the picture upon the ground line at point **3**. Now set off 5′ (*the actual length of* **e f**) from point **3** towards right, and determine the corners of the square **f g**, by joining point **5** to **C V**.

N.B.—Any line drawn upon the ground-plane, parallel to the ground-line, lying between the parallel lines **3 C V, 5 C V**, would be 5′ long, because the actual distance between the points **3** and **5** on the ground-line is 5′, but the farther the line is carried beyond the picture-plane the shorter its representation will appear.

It should be observed that the *diagonals* of the squares converge to **D 1** and **D 2**. *When a square lies upon the ground-plane having two of its edges parallel to the picture-plane its diagonals invariably converge to* **D 1** *and* **D 2**. The student is advised to remember this fact, as it will save a few construction lines in solving other problems.

---

FIG. 2.

I.—Find a point **a** lying upon the ground-plane at 2′ on the spectator's left.

II.—Draw from point **a**, a *vertical* line **a c**, 4′ long, lying in the plane of the picture.

III.—Let the line **a c** be the nearest vertical edge of a square whose surface is in the plane of the picture.

IV. Draw two other squares, equal, parallel to, and immediately opposite, the first square at a distance of 4′ apart.

---

Having prepared the picture-plane, set off 2′ from **X** upon the ground-line towards left, which determines the required point **a**.

Make **a c** vertical and 4′ high, by scale, which represents its actual length, and since the surface of the required square is to be in the plane of the picture we have simply to make a square upon the line **a c** (*See* Obs. XI.).

Set off 4′ on left of point **2**, giving **a b** as the base of the square, then make **b d** vertical, and equal in length to **a c**. Join **c d** which completes the first square.

The required squares are directly opposite and parallel to the first square, therefore, the distance between these squares must be measured on a line *perpendicular* to the planes of the squares and joining their lower corners upon the ground-plane; further, the planes of the squares being *parallel* to the *picture-plane*, the line on which their distances apart are found must necessarily be *perpendicular* to the *picture-plane.*

Join the lower corners, **a**, **b**, of the first square to **C V**, then set off two spaces of 4′ each from point **2**, and join the points, **4**, **4**, to **D 2**, these lines intersect **2 C V**, and give the near lower corners of the two required squares (points **a**. **a**).

Draw lines from **a**, **a**, parallel to the picture-plane to meet **b C V** at points **b**, **b**. Now join the upper corners of the first square to **C V**, and to meet these lines draw verticals from **a**, **a**, **b**, **b**, giving the upper corners of the farther squares at points **c**, **c** and **d**, **d**.

In order to complete the squares draw their upper edges, **c**, **d**, parallel to the ground-line.

---

FIG. 3.

I.—Find three points, **a**, **a**, **a**, lying upon the ground-plane at 2′, 5′

and 8′, respectively, on the right of the spectator and 2′ beyond the picture-plane.

II.—Let these points be the nearest lower corners of three equal squares of 6′ edge, lying in vertical planes *perpendicular* to the picture-plane, each square rests upon the ground-plane on its base. Required their perspective representation.

---

Set off 2′ from **X** along the ground-line towards right, and draw a line from point **2** to **C V**; then, if we set off 2′ on the left of **2**, and join it (point **X**) to **D 1**, it will intersect the former line, **2 C V**, at point **a**, the near corner of the first square.

Determine the other points **a** by setting off 5′ and 8′ respectively from point **X**, then join **5** and **8** to **C V**. Since the points **a**, **a**, **a**, are to be equally distant from the picture-plane, it is obvious that a line joining them must be parallel to the picture-plane, therefore, draw through the first point **a** a line parallel to the ground line to intersect **5 C V**, **8 C V**, at the points **a**, **a**.

Now determine the back corner of the first square at 6′ from point **a**. Set off 6′ from **X**, towards left, then draw a line from point **6** to **D 1**, which gives point **c** on the line **2 C V**.

The line **a c** is the lower edge of the first square, and the bases of the other squares are obtained by drawing a line from corner **c** of the first square, parallel to the ground-line, to meet the lines **5 C V**, **8 C V**, at the points **c**, **c**.

The student must now imagine three vertical planes *behind* and *perpendicular* to the *picture-plane*, each plane to contain one of the required squares.

It has been shown that planes perpendicular to the picture-plane have their intersections with the ground-plane perpendicular to the ground-line (*See* Obs. II., FIG. 12, and Note, FIG. 13). Since the squares lie in similar planes to the above their intersections with the ground-plane must necessarily converge to **C V**.

The first vertical plane passes through the ground-plane in the line **2 C V**, and the intersections of the second and third vertical planes with the ground-plane are shown by the lines **5 C V**, **8 C V**.

These planes are parallel to each other; therefore, they can have but one vanishing line, which is shown by the vertical line drawn through **C V**; their intersecting lines must lie upon the picture-plane at points **2**, **5**, and **8**, because the squares rest upon the ground immediately opposite these points.

N.B.—The vertical plane containing the first square is shaded, which, it is hoped will enable the student to realize its position.

Raise perpendiculars at **a** and **c** in each vertical plane, and to find the perspective height of the first square, set up 6′ from point **2** on **I L** (No. 1), then join point 6′ to **C V**, which determines the height, and its upper edge **a b**.

The height of the second square must be measured on **I L** (No. 2), and that of third square upon **I L**, (No. 3), or, we might determine the upper corners of the second and third squares by drawing from the corresponding corners of the first square parallel to the picture-plane to meet the vertical lines which were drawn from the lower corners **a** and **c**.

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EXERCISE.

Distance of the eye in front of the picture plane, 12′.
Height of the eye above the ground plane, 5′. Scale, ½″ to 1′.

Two points, **A** and **B**, lie on the ground plane at 2′ on the spectator's left; the nearer point, **A**, is 1′ beyond the picture plane, and **A**, **B** are distant from each other 8′. Join **A B**, and upon this line show the perspective representation of a square in a vertical position, the line **A B** being its base.

# PLATE B.

Distance of eye in front of the picture plane, **12′**.
Height of eye above the ground-plane, **9′**.

FIG 1.

A slab is **2′** thick, and has two square faces of **5′** edge. Required, its perspective representation, when resting upon the ground-plane on one of its square faces, two edges of which are parallel to the picture plane, its near corner is **1′** on spectator's left, and **2′** beyond the picture-plane. Upon the upper surface of the slab rests a right pyramid, **5′** high, the base of which is a square of the same size as the slab, having its edges and centre coincident with those of the top surface of the slab.

---

## SECOND POSITION.

Place the same solids in perspective in such a position that the slab rests upon the ground-plane on one of its rectangular faces, its square faces being perpendicular to the picture-plane, and its near corner **5′** on the right of spectator, and **1′** beyond the picture-plane. The apex of the pyramid is the nearest point of the solids to the eye, and the axes of the two solids lie in a vertical-plane parallel to the picture-plane.

---

Set off **1′** from **X**, towards the spectator's left, and join point **1** to **C V**.

Imagine the near edge of the base of the slab to *touch* the picture-plane, in this position it would appear its real length, and is determined by setting off **5′** on the ground-line from point **1** towards the left. Join **5** to **C V**.

We have now to find the near corner of the slab at **2′** beyond the picture-plane. Set off **2′** from point **1**, towards right, and join point **2** to **D 1**, the intersection of this line with **1, C V** gives corner **a** of the slab.

Draw a line from point **a**, towards left, parallel to the picture-line to meet the line **5, C V** at point **b**.

The line **a b** is the perspective length of the near edge of the base of slab when placed at **2′** beyond the picture-plane.

The line **a b** is parallel to the picture-plane, therefore the contiguous sides of the square must be perpendicular to the picture-plane, and lie somewhere upon the lines drawn from **a** and **b** to **C V**; further, because the edge **a b** of the base of slab is parallel to the picture-plane, its diagonals will appear to converge to **D 1** and **D 2**.

Draw a line from **a** to **D 1**, also a line from **b** to **D 2**, these lines intersect those drawn from **5** and **1** to **C V**, at points **c** and **d**, and thus determine the back corners of the square base. Join **c d**.

Erect perpendiculars at **a b c d**, which have to be measured *perspectively* equal to **2′**.

In order to measure the perpendiculars drawn from **b** and **c**, it is necessary to imagine a vertical plane to contain the whole side of the slab of which these perpendiculars are two edges. This vertical-plane intersects the ground-plane in the chain-line passing through **b c**; it likewise cuts the picture-plane in the vertical line drawn through point **5** (**I L** No. 1).

PLATE B.

FIG. 1.

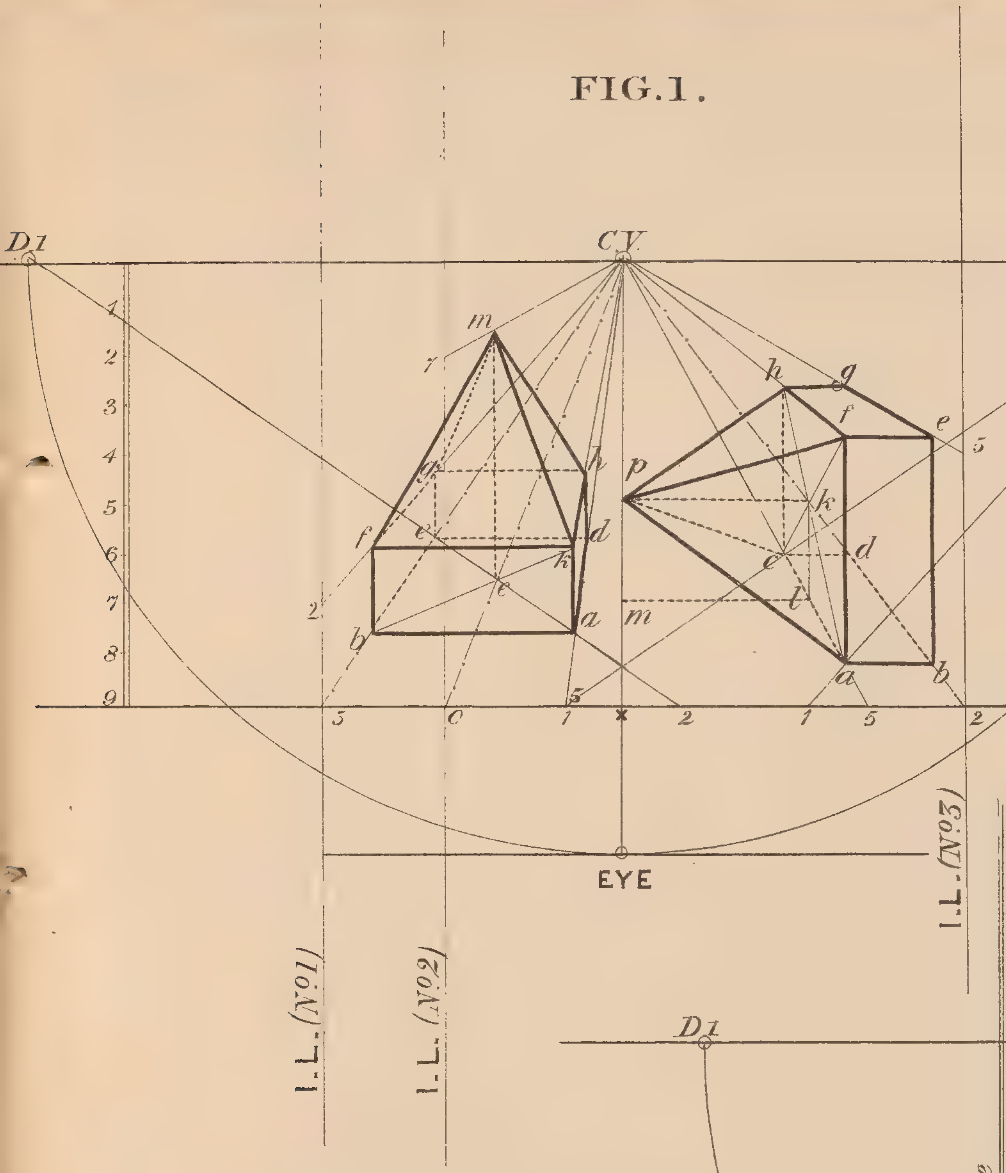



FIG. 2.

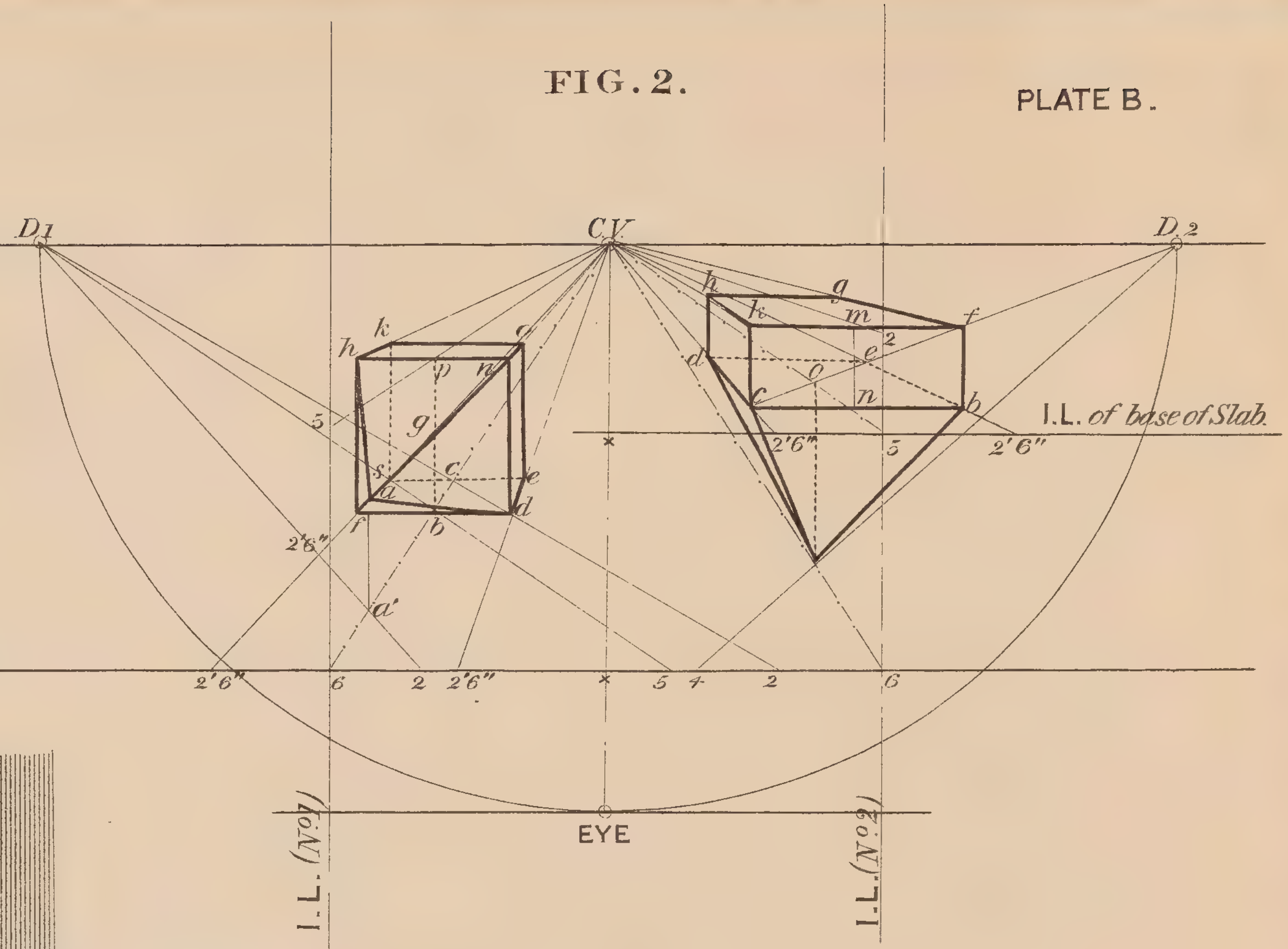
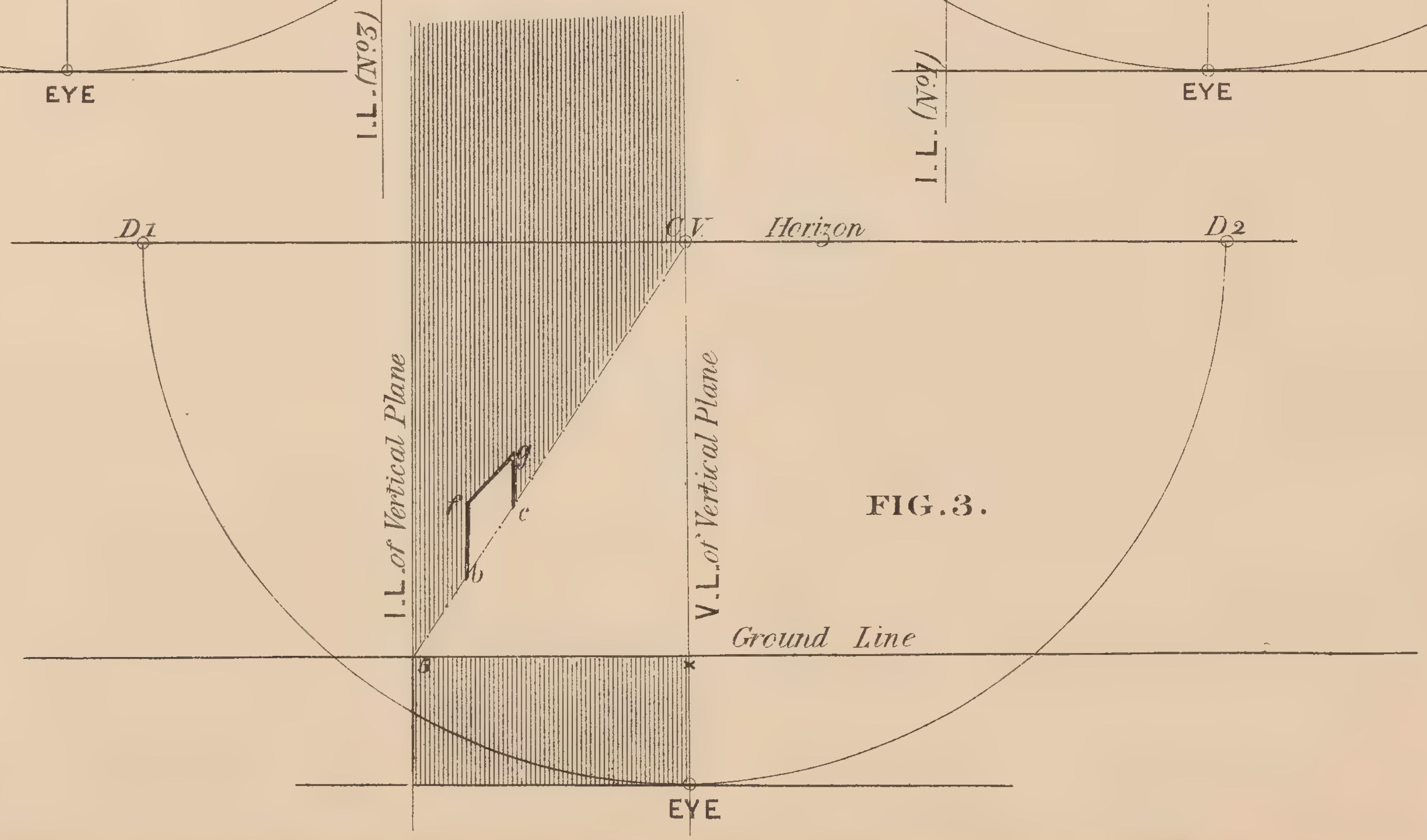



FIG. 3.

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N.B.—A separate sketch is given (FIG. 3, PLATE B), showing the vertical plane containing the side of the slab. It is shaded, that the student may more clearly understand its position and intersections with the ground and picture planes.

Upon **I L** (No. 1) set up **2′** from point **5**, draw a line from point **2** to **C V**, which intersects the perpendiculars drawn from **b c**, and gives an edge (**f g**) of the upper surface of the slab.

The perpendiculars drawn from **a b**, are at an equal distance from the picture-plane and *actually* equal in length, therefore their perspective lengths will appear equal. Again, because the lower edge (**a b**) of the slab is parallel to the picture-plane, the upper edge immediately above it is also parallel, therefore draw **f k** parallel to **a b**. The nearest surface of the slab is shown by the rectangle **a b f k**.

Draw a line from **f** to **C V**, which intersects the perpendicular from **c** at point **g**; also draw a line from **k** to **C V**, giving point **h** on the perpendicular drawn from **d**. Join **g h**, which completes the representation of the slab.

Now find the centre **e**, of the base of the slab, by drawing the diagonals **b d**, **a c**, and in order to measure the axis of the pyramid we must imagine a vertical plane, *perpendicular* to the picture-plane, to contain it.

The chain-line drawn from **C V** through **e**, and produced to meet the ground-line at point **o**, is the intersection of this vertical plane with the ground-plane.

Through point **o** draw **I L** (No. 2), perpendicular to the ground-line, and upon it, from **o**, set up **7′**, which represents the actual height of the apex of the pyramid above the ground-plane.

Erect a perpendicular at point **e**. Join point **7** to **C V**, which determines the apex of the pyramid at point **m**; then to complete the second solid we have simply to join **m** to each of the upper corners of the slab.

---

## SOLUTION OF SECOND POSITION (FIG. 1).

Upon the ground-line, towards right, set off **5′** from **X**, and join point **5** to **C V**. From **5** set off **1′** to left, and join point **1** to **D 2**, the lines so drawn intersect each other at point **a** and give the nearest corner of the slab upon the ground plane.

Set off **2′** from point **5**, towards right, and join point **2** to **C V**, then draw a line from **a** parallel to the ground-line to intersect the line **2 C V** at point **b**.

**a b** is the nearest edge of the rectangular slab upon the ground-plane, and is parallel to the picture-plane; the adjacent edges to **a b** are perpendicular to it, and consequently perpendicular to the picture-plane; their perspective representation will be found upon the lines joining points **5 2** to **C V**.

Point **a** is **1′** beyond the picture-plane, therefore set off **5′** from point **1**, towards left, and join **5** to **D 2**, giving **a c** the second edge upon the ground-plane.

Draw **c d** parallel to **a b**, which completes the rectangular face.

Erect a perpendicular at each corner of the rectangle, and measure those drawn from corners **b d**.

These perpendiculars lie in a vertical-plane, perpendicular to the picture-plane, whose intersection with the ground-plane is the chain-line **2 C V**, and its intersection with the picture-plane is the vertical-line (**I L** No. 3) drawn through point **2**.

Upon **I L** (No. 3) set up **5′** from **2**, and join **5** to **C V**, which gives **b d g e** as the farther square face of the slab. The nearer square face is determined by drawing **e f**, **g h**, parallel to **a b**, to meet the verticals drawn from **a c**.

Determine the centre, **k**, of the nearer square, by drawing the diagonals **a h**, **c f**.

Point **k** is the farther extremity of the axis of the pyramid. The axis of the pyramid lies above the ground; therefore cannot be measured on that plane.

There are two methods by which we can obtain the perspective length of the axis of the pyramid.

1st.—By finding a horizontal plane to contain it, and measuring it in this plane by **C V**.

2nd.—By obtaining a line upon the ground-plane perspectively equal to, and immediately below, the axis, and by making the apex of the pyramid immediately above the nearer extremity of the line upon the ground-plane.

We will proceed to measure the axis by the 2nd method. Let fall a perpendicular from **k** to meet the ground-plane at **l**, upon the intersection of the nearer square face of slab with the ground (line **a c**). Through **l** draw a line parallel to the ground-line, which has to be measured equal to 5′.

Bring forward **l** by **C V** to meet the ground-line at point **5**. From **5** set off **5′** towards left (point **X**), then join **X** to **C V**, giving **l m** perspectively equal to **5′**.

Through **k** draw a line parallel to **l m**, and determine the apex of the pyramid by making **p** vertically over point **m**.

Join **p a**, **p c**, **p f**, **p h** for the second solid.

---

FIG. 2.

Distance of eye in front of the picture-plane, **12′**.

Height of eye above the ground-plane, **9′**.

Required the perspective representation of the slab and pyramid in the last exercise, in the following positions:

I.—When the slab rests upon the ground-plane on one of its rectangular faces, and the axes of the two solids lie in a vertical plane perpendicular to the picture-plane, at **6′** on the spectator's left. The apex of the pyramid to be **2′** beyond the picture-plane.

II.—When the pyramid is balanced upon the ground-plane on its apex, and the axes of the two solids lie in a vertical plane, perpendicular to the picture-plane at **6′** on the spectator's right. The apex of the pyramid is **4′** beyond the picture-plane, and two rectangular faces of the slab are parallel to the picture-plane.

---

## SOLUTION OF FIRST POSITION (FIG. 2).

It is necessary to find the vertical plane perpendicular to the picture-plane, at **6′** on the spectator's left, containing the axes of the two solids.

Set off **6′** from **X**, *towards left*, and join point **6** to **C V** by a chain-line.

The chain-line, **6 C V**, represents the intersection with the ground plane of the vertical plane containing the axes.

Through point **6** draw **I L** (No. 1), which represents the intersection of the above-mentioned vertical plane with the picture-plane, and since this vertical-plane passes right through the centres of the solids, it is obvious that one-half must be represented to the right and the other to the left of it.

Upon the ground set off **2′ 6″** on right and left of point **6**, and join the points to **C V**.

Now, find the position of a point lying upon the ground-plane, immediately under the apex of the pyramid, also a line from it, lying upon the ground, vertically below the axis of the pyramid.

Set off **2′** from **6** towards right, and join point **2** to **D 1**, which determines **a′** (*the point under the apex*) on the chain-line, **6 C V**. Again, set off **5′** (*length of axis of pyramid*) from **2′** towards right, and join point **5** to **D 1**, giving **a′ b** perspectively equal to, and vertically below, the axis of the pyramid.

Set off **2′** from **5**, towards right, and join point **2** to **D 1**, this line intersects chain-line **6 C V** at point **c**, the line **b c** is therefore the intersection of the vertical plane, containing the axes, with the rectangular face of the slab which lies upon the ground-plane.

Through points **b c**, draw lines parallel to the ground-line to meet the lines drawn from the points **2′ 6″** to **C V**, at **f d e s**.

Set off **5′** from **6** on **I L** (No. 1), and join point **5** to **C V**, then raise a perpendicular at **b** to meet **5 C V**, at point **p**. The chain-line **b p** is the intersection of the vertical plane with the nearer square face of the slab.

Through point **p** draw a line parallel to **f d**, and at **f d** erect perpendiculars to meet the former line at **h n**.

**f d h n** is the near face of the slab.

The axis of the pyramid is 2′ 6″ above the ground-plane; therefore set up 2′ 6″ from **6** on **I L** (No. 1), and join it to **C V**, giving **g**, the centre of the square face of slab, also the farther extremity of the axis.

Erect a perpendicular at **a′** to meet the line **g C V** produced, at point **a**, which is the perspective representation of the apex of the pyramid.

Join **a f**, **a d**, **a h**, **a n**, for the inclined edges of the pyramid.

We have now to determine the thickness and back surface of the slab.

At **e s** erect perpendiculars, and to intersect them at points **k o**, draw lines from **n h** to **C V**.

Lastly, join **o k** for the upper edge of the back surface of the slab.

## SOLUTION OF SECOND POSITION (FIG. 2).

Find the vertical plane perpendicular to the picture-plane which contains the axes of the solids at 6′ on the spectator's right.

Set off 6′ from **X** towards right, and join point **6** to **C V** by a chain-line.

The chain-line **6 C V** is the intersection of the vertical plane of the axis with the ground-plane, and since the pyramid rests upon its apex, it is obvious that its perspective representation will be found upon this chain-line.

Upon the ground-line set off 4′ from **6** towards left, and join point **4** to **D 2**. The line **4 D 2** intersects the chain-line **6 C V**, and gives point **a** as the apex of the pyramid.

The pyramid is 5′ high, therefore the horizontal plane which contains its base will intersect the plane of the picture at 5′ above, and parallel to the ground-line.

Set up 5′ from **6** on **I L** (No. 2), through point **5** draw **I L** of lower surface of slab and base of pyramid, parallel to the ground-line.

The intersection of the plane of base of pyramid with the vertical plane of the axes is shown by the chain-line **5 C V**, and the upper extremity of the axis of pyramid will be determined upon it by drawing the perpendicular **a o**.

Two edges of the base of the pyramid are to be parallel to the picture. It has been previously shown that a square in such a position always has its diagonals converging to **D 1** and **D 2**; therefore, through point **o** draw lines to **D 1**, **D 2**, and the corners of the base will be determined upon these lines.

Upon **I L** of plane of lower surface of slab set off 2′ 6″ from **5**, towards right and left; then, by joining the points 2′ 6″ to **C V**, the corners **c b**, **d e** are found.

We have now to determine the thickness and upper surface of the slab. Set up 2′ from **5** on **I L** (No. 2) and join point **2** to **C V**.

The vertical plane of the axes cuts the nearest edge of the base of the pyramid at point **n**; therefore, at **n** draw a perpendicular to meet **2 C V**, at **m**.

Point **m** is the centre of the upper edge of the slab, and the chain-line **n m** is the intersection of the vertical plane of the axes with the nearest vertical face of the slab.

Through point **m** draw a line parallel to the ground-line, and to meet it at **f k**, draw perpendiculars at **b c**. Now join **f k** to **C V**, and to meet these lines at **h g**, draw vertical lines from the back corners **d e**.

Complete the upper surface of the slab by joining **h g**.

## EXERCISE.

Distance of eye in front of the picture plane, 12′.
Height of eye above the ground plane, 5′. Scale, ½″ to 1′.

A square of 6′ side lies upon the ground plane, its nearest side is parallel to the picture plane and 3′ beyond it, and two of its corners are directly opposite to the spectator. This square is the base of a right prism 10′ high; complete its perspective representation.

# PLATE C.

Distance of eye in front of the picture-plane, 12′.
Height of eye above the ground-plane, 9′.

FIG. 1.

I.—Draw in perspective a circle 6′ in diameter lying in a verticalplane parallel to the picture-plane, its centre being 5′ on spectator's left, and 4′ beyond the picture-plane. Draw a second circle of equal diameter, parallel to, and directly opposite the first circle, and 8′ distant from it. These circles are the ends of a right cylinder. Complete the representation of the solid.

II.—Required the perspective representation of a circle 6′ diameter, lying upon the ground-plane, its centre being 5′ on the right of the spectator, and 4′ beyond the picture-plane. A second circle of equal diameter is parallel to and immediately above the first circle, and 6′ distant from it. These circles are the ends of a right cylinder; complete the solid.

## SOLUTION OF FIRST POSITION (FIG. 1).

Because the planes or *surfaces* of the circles are vertical and parallel to the picture-plane, their representations are *circles*, and a vertical plane containing their centres must necessarily be *perpendicular* to the plane of the picture.

We will proceed to find the vertical plane perpendicular to the picture-plane, containing the centres of the circles, by drawing its intersection with the picture-plane (**I L** No. 1) vertically, at 5′ from point **X**, towards *left*; its intersection with the ground-plane is determined by drawing a chain-line from point **5** to **C V**, and the points of contact of the circles will be found upon this chain-line immediately below the centre of each circle.

Upon the ground-line set off 4′ from **5**, towards right; then join point **4** to **D 1**; the intersection of this line with the chain-line, **5 C V**, gives the point of contact, **a**, of the *first* circle. Now, set off the actual distance between the circles, viz., 8′, from **4** towards right, and by joining point **8** to **D 1**, the *second* point of contact, **b**, is found.

At **a b** draw vertical lines, which have to be measured *perspectively* equal to 3′.

Upon **I L** (No. 1) set up 3′ from **5**, and join point **3** to **C V**. The intersection of the line **3 C V** with the verticals drawn from **a b** gives **c** and **d** as the centres of the required circles.

From the centre **c**, with the radius **c a**, describe the nearer circle; again, from **d** as centre, with radius **d b**, describe the farther circle.

In order to complete the representation of the cylinder, draw two straight lines tangential to the circles, and converging to **C V**.

## SOLUTION OF SECOND POSITION (FIG. 1).

Find a vertical plane perpendicular to the picture-plane containing the centres of the required circles at 5′ on the spectator's right.

The vertical line **I L** (No. 2) is its intersection with the picture-plane and the chain-line **5 C V** is its intersection with the ground-plane.

Upon the ground-line set off 4′ from **5** towards left, and join point **4** to **D 2**, giving point **a**, the centre of the circle upon the chain-line **5 C V**.

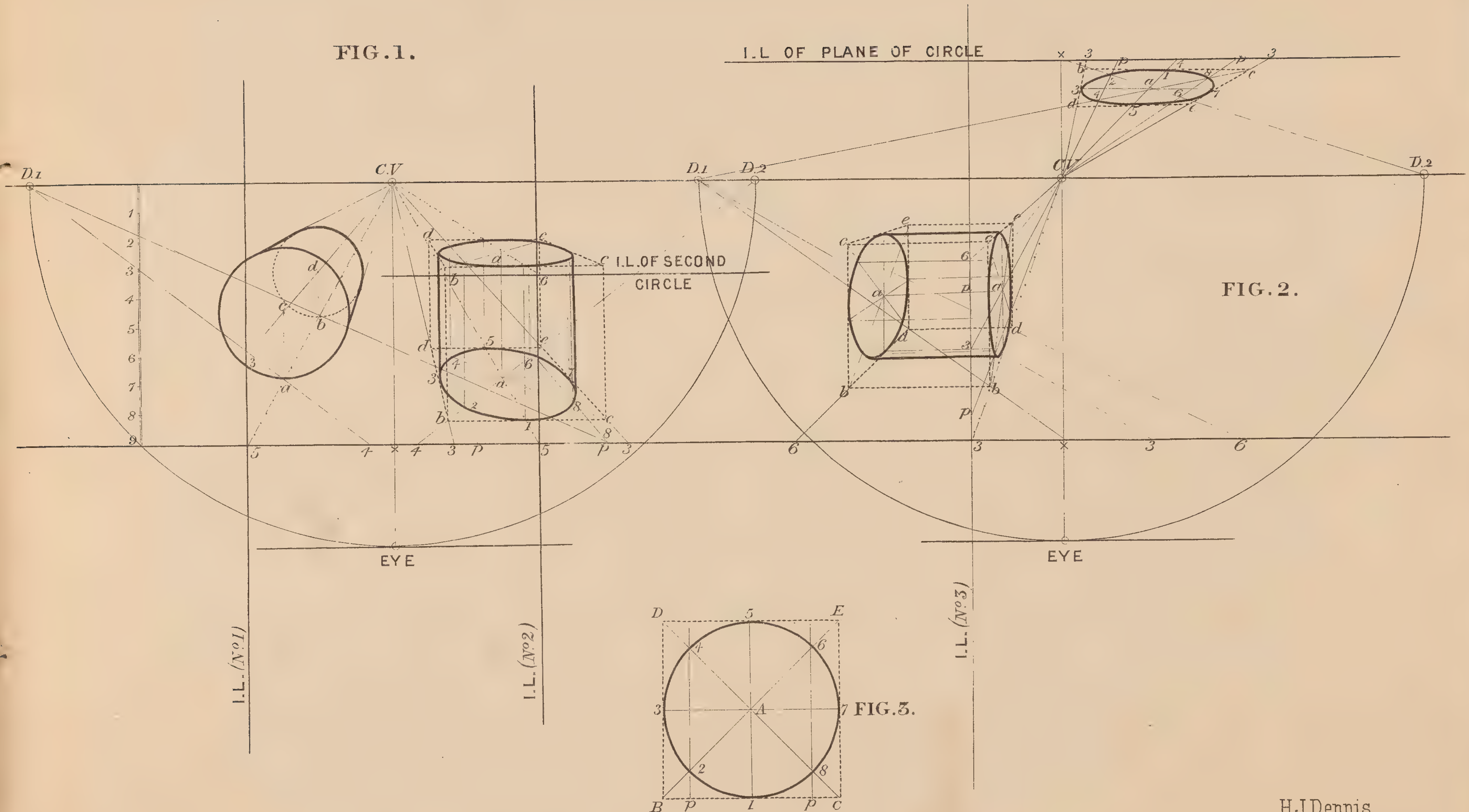


FIG. 1.

FIG. 2.

FIG. 3.

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N.B.—Before the circle can be obtained in perspective, it is necessary to draw its *plan*, 6′ diameter, upon any convenient part of the paper, and enclose it within a square; then draw its diagonals and diameters so as to divide the circumference of the circle into eight equal parts (*See* FIG. 3), and draw lines parallel to the side of the square (**B D**) through the points **2 4, 6 8**.

We will now proceed to find the perspective representation of the circumscribing square, and the eight points within it upon the ground-plane.

Having found the centre **a** of the square, draw lines through it to **D 1** and **D 2**; the diagonals will be situated somewhere upon these lines.

Upon the ground-line set off 3′ on right and left of point **5**, then draw a line from **3 3** to **C V**, giving the four corners of the square at points **b c d e**. Join **b c d e**.

*The square is dotted that it may be better distinguished from the construction lines.*

Draw the diameters, **3 7, 1 5**, through point **a**, the former parallel to **c**, and the latter converging to **C V**.

Now measure the distances **1 p, 1 p** (FIG. 3); set them off on the ground-line on right and left of point **5**, and join the points **p p** to **C V**; these lines intersect, and give points **2 4 6 8** upon the diagonals.

Through the eight points within the square draw the representation of the first circle.

---

## METHOD OF FINDING THE SECOND CIRCLE.

Since the plane of the second circle is 6′ above the ground-plane, its **I L** must be 6′ above the ground-line; therefore upon **I L** (No. 2) set up 6′ from the ground-line, and through point **6** draw the **I L** of the plane of the second circle parallel to the ground-line.

The intersection of the plane of the second circle with the vertical plane containing the centres of the circles is shown by the chain-line **3 C V**, and the centre of the upper circle is determined upon this chain-line by drawing a vertical line from the centre of first circle.

The second circle is equal in diameter to the first, and lies vertically over it; therefore the two circumscribing squares must be vertically over each other, and the diagonals and sides of each square are parallel.

Through the centre **a** of the second circle, draw the diagonals of its circumscribing square to **D 1, D 2**, because those of the first square converge to these points; then raise perpendiculars from the corners of the first square to meet the above diagonals, which give the four corners of the second square. Draw its diameters, one parallel, the other perpendicular to the picture, the extremities of which are four points in the required circle, and the remaining points are found by drawing perpendiculars from the points on the diagonals of the first square (*points* **2 4, 6 8**) to intersect the corresponding diagonal of the second square.

Through the eight points thus obtained, draw the representation of the second circle.

In order to complete the cylinder, we have simply to draw its vertical sides tangential to the elliptical ends.

*N.B.—In finding the representation of a cylinder, or prism of any shape, the student is advised to imagine it enclosed within a square or rectangular prism, as the case may be.*

*In this plate the cylinder is enclosed within an imaginary square prism, and shown by the dotted lines.*

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Distance of eye in front of the picture-plane, **12′**.
Height of eye above the ground-plane, **9′**.

FIG. 2.

**I.**—Draw in perspective a circle 6′ diameter lying in a vertical plane perpendicular to the picture-plane, the nearest point in its circumference being 3′ on spectator's left, and 3′ beyond the picture-plane. Draw a second circle of equal diameter parallel to, and directly opposite the first circle, and 6′ distant from it towards left. Complete the perspective representation of a cylinder, the circles being its ends.

II.—Required the perspective representation of a circle 6′ diameter, lying in a horizontal-plane 4′ *above* the horizon, its centre being 4′ on right of spectator, and 4′ beyond the picture.

SOLUTION OF FIRST POSITION (FIG. 2).

Obtain the vertical plane of the first circle by drawing **I L** (No. 3) at 3′ from point **X**, towards left. This vertical plane intersects the ground-plane in the chain-line **3 C V**.

Since the nearest point in the circumference of the circle is to be 3′ beyond the picture, it is evident the nearest corner of the circumscribing square must be at the same distance from the picture, because the two points lie immediately over each other.

Join **X** to **D 1**, which gives the near corner **b** on the chain-line **3 C V**. The farther corner **d** is obtained by setting off 6′ (*actual side of square*) from point **X**, towards right, and joining point **6** to **D 1**.

Upon **I L** (No. 3) from point **3** set up the distances **3 p, p 3, 3 p, p 6**, respectively equal to the distances **B p, p 1, 1 p, p C** (FIG. 3), and join the points to **C V**.

Erect perpendiculars at **b d**; where these lines intersect **6 C V** the upper corners of the square are obtained.

Draw the diagonals and diameters; the extremities of the diameters are four points in the elliptical end, and the intermediate points are obtained at the intersections of the lines **p C V, p C V** with the diagonals.

Through the eight points thus determined, draw the representation of the first circle.

Now set off 6′ from **3**, towards left, and join point **6** to **C V**. Through point **b** draw a line parallel to the ground-line to meet **6 C V** at point **b**.

The line **b b** represents the lower edge upon the ground of the imaginary circumscribing square prism.

Draw **d d** parallel to **b b**; at **b d**, draw verticals upon which the upper corners of the second square are found by drawing **c c, e e**, parallel to **b b**.

Draw the diagonals and diameters of the second square, and determine the points upon its diagonals by drawing lines from the points on the diagonals of the first square parallel to **b b**.

Through the points draw the representation of the second circle.

The sides of the cylinder are found by drawing tangents to the elliptical ends; and these tangents must necessarily be parallel to the picture-plane, because the ends are perpendicular to that plane.

SOLUTION OF SECOND POSITION (FIG. 2).

Draw the **I L** of plane of circle at 4′ above, and parallel to the *horizon*.

Find point **X** immediately over **C V**, and from it set off 4′, towards right. Join point **4** to **C V**; also join point **X** to **D 2**, the intersection of these lines (point **a**) is the centre of the required circle.

On **I L** of plane of circle set off **4 p, 4 3**, on right and left of point **4** respectively equal to **1 p, 1 B** (FIG. 3).

Join **p 3, p 3**, to **C V**. Through point **a**, draw the diagonals of the circumscribing square to **D 1, D 2**, giving **b c, d e** on the lines **3 C V**. The intersection of the line **4 C V** with the front and back edges of the square gives the diameter **1 5**; the other diameter is obtained by drawing through **a** parallel to horizon.

The intermediate points are found at the intersections of the diagonals with the lines **p C V, p C V**, at **2 4 6 8**.

Draw the representation of the circle through the points thus found.

EXERCISE.

Distance of eye in front of the picture plane, 12′.
Height of eye above the ground plane, 5′. Scale, ½″ to 1′.

The base of a right cone is a circle of 6′ diameter, and its axis is 8′ long. Give the perspective representation of this solid when its base is *vertical* and *parallel* to the picture plane, and in contact with the ground plane at a point 1′ on the left of the spectator and 10′ beyond the picture plane. The apex of the cone is the nearest point of the solid to the picture plane.

PLATE D.

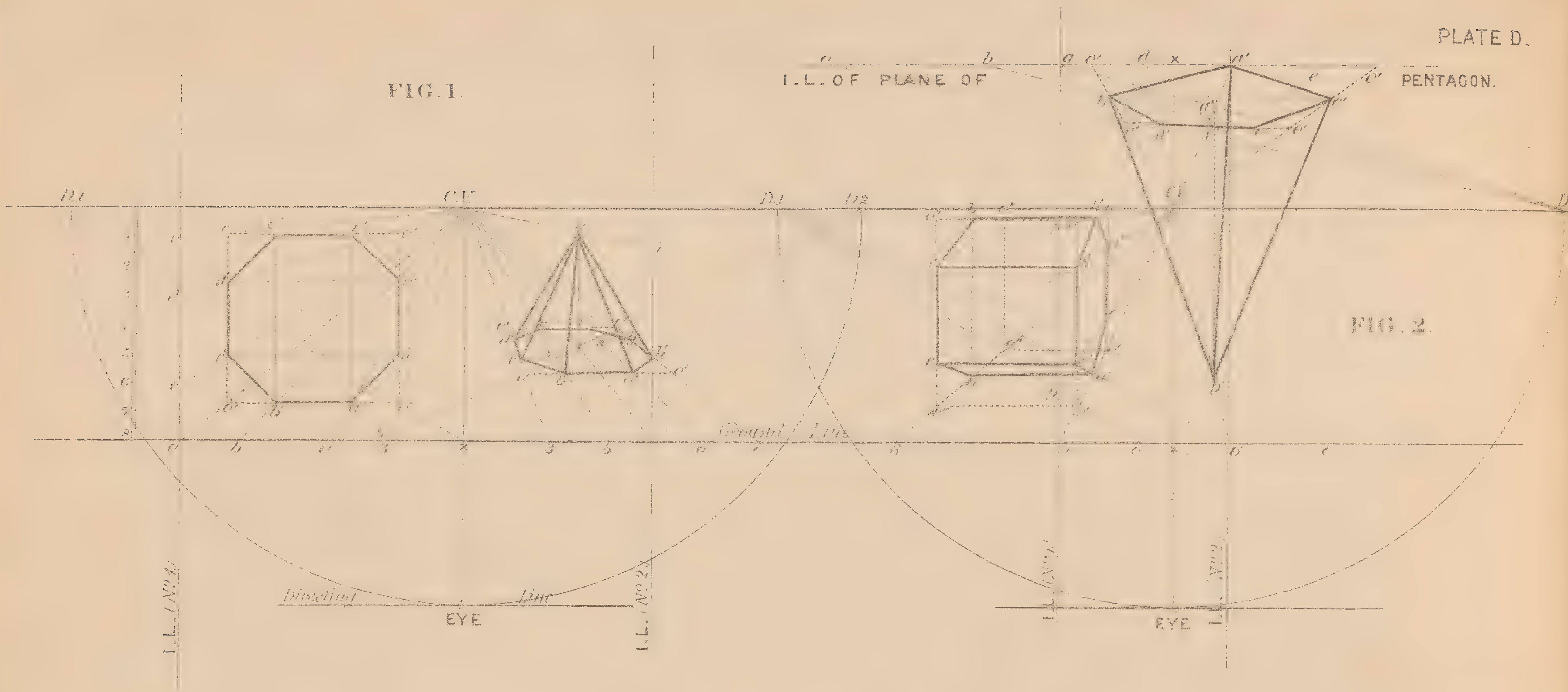


FIG. 1.

FIG. 2.

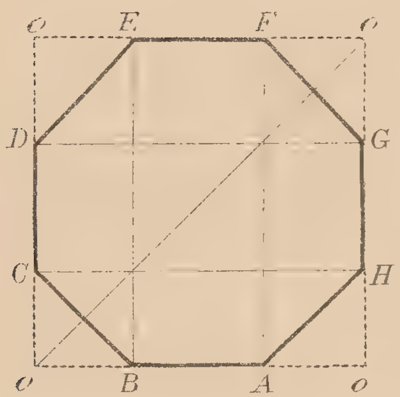


FIG. 3.

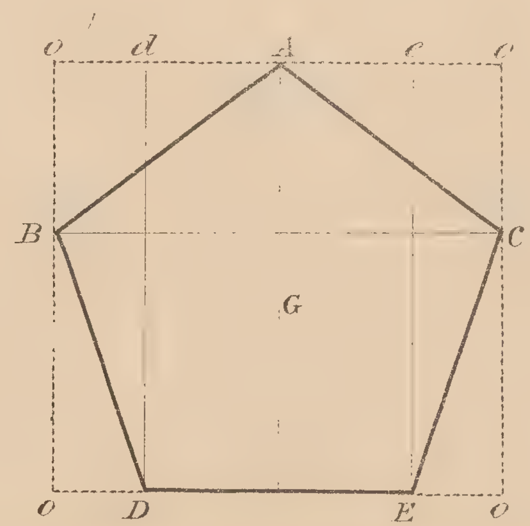


FIG. 4.

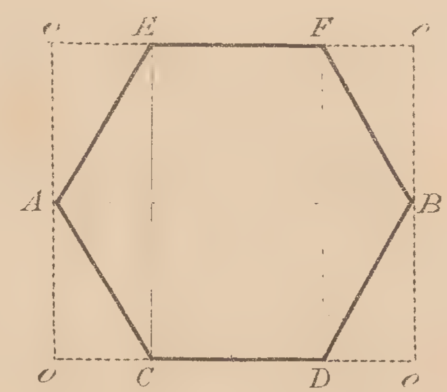


FIG. 5.

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# PLATE D.

Distance of eye in front of the picture-plane, **14′**.
Height of eye above the ground-plane, **8′**.

FIG. 1.

I.—Draw in perspective a regular octagon of **3′** edge, its plane (*surface*) being vertical and parallel to the picture plane; it rests upon the ground on one of its edges, and two others are vertical, the nearer of which is **3′** on left of spectator, and **3′** beyond the picture-plane.

II.—A right* octagonal pyramid, **3′** edge of base, **7′** high, rests upon the ground-plane on its base, the nearest edge of which is parallel to the picture-plane and **6′** beyond it; another of its edges is perpendicular to the picture-plane, and **3′** on the right of the spectator.

* *A right pyramid, or prism, has its axis perpendicular to its base.*

## SOLUTION OF FIRST POSITION (FIG. 1).

*N.B.—In order to find the perspective representation of any polygon, its plan should be drawn and circumscribed by a parallelogram, so that the corners of the former are situated in the sides of the latter.*

*The representation of the circumscribing parallelogram and the points (corners of the polygon) upon its sides should be next determined, and if these points be joined the polygon will be found.*

Construct the plan of the octagon upon a line **A B** equal to **3′**, by scale, and circumscribe it by a *square* (*See* FIG. 3). The alternate sides of the octagon are produced, and a square is the result.

The circumscribing square has now to be determined, its near corner upon the ground-plane lies immediately below the nearer vertical edge of the octagon, and consequently must be found at **3′** on the left, and **3′** beyond the picture-plane.

Upon the ground-line set off **3′** from **X**, towards left, and join point **3** to **C V**, also join point **X** to **D 1**, the point of intersection, **O′** is the near corner of the square upon the ground-plane.

Again, set off **3 O** equal in length to side of the circumscribing square, and join point **O** to **C V** by a chain-line.

From **O′** draw a line parallel to the picture-plane, towards left, to meet the chain-line **O C V** at **O′**. The dotted line **O′ O′** is the base of the circumscribing square.

The student must now imagine a vertical plane *perpendicular* to the picture-plane, to contain either of the vertical edges of the square; say that farther from the eye, because its **I L** will be situated upon the picture-plane away from the other construction lines, and render the work more intelligible.

The chain-line **O C V** is the intersection with the ground of the above-mentioned plane, and **I L** (No. 1) is its intersection with the plane of the picture.

Upon **I L** (No. 1) set up **O O** equal to **O O** (*side of square*), FIG. 3. Join the upper point **O** to **C V**; then from point **O′** on the chain-line **O C V** draw a vertical dotted line to meet the upper line **O C V** at point **O**.

Complete the circumscribing square, by drawing its upper edge parallel to the ground-line, and the remaining edge parallel to the vertical edge previously obtained.

Having completed the square, we will proceed to obtain the representation of the corners of the octagon upon its edges.

Measure the distances **O A, A B** (FIG. 3), and make **3 a, a b**, upon the ground-line, respectively equal to them. Join **a b** to **C V**, intersecting the base of the square at **a′ b′**, and thus giving the base of the octagon.

Measure **O C, C D** (FIG. 3), and make **o c, c d**, on **I L** (No. 1) equal to them. Draw lines from to **c d** to **C V**, which intersect and determine two other corners of the octagon upon the vertical side of square at **c′ d′**.

By referring to FIG. 3, it will be seen that corners **G H** lie immediately opposite **D C**, in lines parallel to **A B**; therefore, if lines be drawn from **c′ d′** parallel to **a′ b′** to meet the opposite vertical edge of the square, we shall have corners **g′ h′** of the octagon.

The remaining corners are found by drawing vertical lines from **a′ b′** to meet the upper edge of the square at points **e′ f′**, and to complete the octagon the corners **a′ h′, g′ f′, e′ d′, c′ b′**, must be joined.

---

## SOLUTION OF SECOND POSITION (FIG. 1).

*One side of the octagon is to be* **3′** *on the right of the spectator; therefore make one side of the circumscribed square at* **3′** *on right.*

*Another side of the octagon is to be* **6′** *beyond, and parallel to the picture-plane; therefore, let the representation of the near side of the circumscribing square occupy a like position.*

Upon the ground-line set off **3′** from point **X** towards right, and join **3** to **C V**; then determine the near corner of the square by setting off **6′** from **3**, and joining point **6** to **D 2**.

Again, upon the ground-line set off **3 O** equal to **O O** (side of square), FIG. 3. Join **O** to **C V**; then draw a dotted line from the near corner **O′** parallel to the picture-plane to meet **O C V** at **O′**.

*Because the near side of the square is parallel to the picture-plane, its diagonals converge to* **D 1, D 2**.

Join the near corners **O′ O′** to the opposite points **D 1, D 2**, giving the back corners **O′ O′** upon the lines **3 C V, O C V**.

Having found the square, the corners of the octagon have to be determined upon its sides, and by referring to FIG. 3, it will be seen that corners **E F** lie on the farther edge of the square, immediately opposite to **B A**.

Upon the ground-line set off the distances **3 b, b a**, respectively equal to **O B, B A**, FIG. 3. Draw lines from **b a** to **C V**; these lines intersect the front and back edges of the square at points **b′ a′, e′ f′**, and give four corners of the required octagon.

Now, if a diagonal (FIG. 3) be drawn, it will be found to intersect **B E, A F**, in points which are immediately opposite the remaining corners of the octagon; therefore, if we draw through points **s s** in our perspective representation parallel to **a′ b′**, to meet the sides of the square at points **d′ c′, g′ h′**, we shall have the representation of the remaining four corners of the octagon.

Join **d′ e′, f′ g′, h′ a′, b′ c′**.

A vertical plane perpendicular to the picture, containing the axis of the pyramid is now required.

The chain-line drawn through **1** to **C V** is its intersection with the ground-plane, and **I L** (No. 2) is its intersection with the picture-plane.

Set up **7′** from the ground on **I L** (No. 2); then, by joining **7** to **C V**, the apex of the pyramid will be determined at point **k**.

Lastly, join **k** to each corner of the octagon for the inclined edges of the pyramid.

---

Distance of eye in front of the picture-plane, **14′**.
Height of eye above the ground-plane, **8′**.

## FIG. 2.

I.—A right prism, **6′** long, has regular hexagonal ends of **3½′** edge. It rests upon the ground-plane on one of its **6′** edges, which is parallel to the picture-plane, and **6′** beyond it. The planes of its ends are vertical, and perpendicular to the picture-plane, and the nearer end is **4′** on the left of the spectator. Two of the rectangular faces of the prism are perpendicular to the ground-plane.

II.—A right pyramid has a regular pentagon of **6′** edge for its base which lies in a horizontal-plane **5′** above the horizon (**13′** *above the ground plane*); one of its corners is in contact with the picture-plane at **2′** on the

right of the spectator, its farthest edge is parallel to the picture-plane, and its apex is in contact with the ground-plane. Give its perspective representation.

## SOLUTION OF FIRST POSITION (FIG. 2).

Upon a line 3½′ long, construct a regular hexagon to represent the end elevation of the prism (*See* FIG. 5). Describe a rectangle about the hexagon by producing the sides **E F**, **C D**, to meet perpendiculars drawn through **A**, **B**, at points **O O O O**.

Find the vertical plane which contains the nearer end of the prism. Set off 4′ from **X**, towards left, and through point 4, draw **I L** (No. 1). The intersection of this vertical plane with the ground is determined by the chain-line joining 4 to **C V**.

Upon the ground-line set off 6′ from point 4 towards right, then draw a line from 6 to **D I**, which intersects chain-line 4 **C V** at a′, the near extremity of the edge of the prism which lies upon the ground-plane.

Again, set off 6′ from point 4, towards *left* and join 6 to **C V**, then draw the edge of the prism a′ a parallel to the ground-line to meet 6 **C V**.

On right and left of point 6 set off 6 **O**, 6 **O**, equal to **A O**, **A O**, FIG. 5. Join the points **O O** to **D I**; these lines intersect 4 **C V** at O′, O′, and give the representation of the side of the circumscribing rectangle.

Upon **I L** (No. 1) set up 4 p, p p, p p equal to **O E**, **E F**, **F O**, FIG. 5. Join the points p p p to **C V**, and to meet these lines draw verticals at o′, a′, o′.

In order to complete the representation of the near end of the prism we have simply to join the points of intersection e′ a′, a′ c′, f′ b′, b′ d′.

Draw lines parallel to the ground-line from the four corners of the rectangle which circumscribes the near end of prism; the lengths of the lines upon the ground are determined by the line 6 **C V**, while those above are determined by vertical lines drawn from O″ O″.

O″ O″ O″ O″ is the representation of the rectangle about the farther end of the prism.

Complete the prism by drawing e′ e, f′ f, b′ b, parallel to a′ a. Join e a, f b.

## SOLUTION OF SECOND POSITION (FIG. 2).

Draw the **I L** of plane of base of pyramid parallel to the *horizon* and 5′ above it.

Find point a′, the near corner of the base of the pyramid, at 2′ from **X** towards right.

Upon the **I L** of the plane of the pentagon set off on right and left of a′ the distances a′ o′, a′ o′, equal to **A O**, **A O**, FIG. 4, and join the points to **C V**.

Again, from a′ set off a′ d, a′ e, equal to **A** d, **A** e, FIG. 4, and join the points to **C V**.

Upon **I L** of pentagon set off the distances o′ b, b o equal to **O B**, **B O**, FIG. 4, and join b o to **D 2**, giving b′ one corner of the pentagon on the dotted line O′ **C V**, also one of the back corners of the circumscribing rectangle, on the same line, at point O′, through which draw the farther edge of the rectangle, parallel to the **I L** of pentagon.

Draw b′ c′ parallel to the ground-line. The lines d **C V**, e **C V** intersect the farther edge of rectangle at d′, e′, which are the remaining corners of the pentagon.

Join a′ b′, a′ c′, c′ e′, e′ d′, d′ b′.

We have now to find a vertical plane perpendicular to the picture-plane which will contain the axis of the pyramid.

This plane will contain the near corner (a′) of the base of the pyramid; therefore, draw a chain-line from a′ to **C V**, representing the intersection of the plane of the axis with the plane of the pentagonal base. Its intersection with the picture plane is the line **I L** (No. 2), drawn through point a′, and its intersection with the ground is the chain-line 6 **C V**.

Upon **I L** of plane of pentagon set off a′ g equal to **A G**, FIG. 4. Join g to **D 2** giving g′ the centre of the pentagon, from which the axis must be drawn.

Let fall a perpendicular from g′ to meet the ground upon the chain-line 6 **C V** at point p′, which represents the apex of the pyramid.

Join p′ to each corner of the pentagon for the inclined edges of the solid.

# PLATE E.

Distance of eye in front of the picture-plane, 12′.

Height of eye above the ground-plane, 5′.

I. A slab 12′ square, 1′ 6″ thick, lies upon the ground-plane on one of its square faces; its centre is immediately opposite the spectator, and one of its rectangular faces is parallel to, and 2′ beyond, the picture-plane.

II. Two other square slabs rest upon the upper surface of the first slab; their square faces lie in vertical planes perpendicular to the picture-plane, each slab rests upon a rectangular face, forming its thickness. These slabs are 8′ square, 1′ thick, and rest in such a position that there is a space of 6′ between them. There is also a space of 2′ between the outer edges of their rectangular bases and the edges of the upper surface of the first slab.

III. Upon the two smaller slabs rests a fourth, equal in dimensions to, and immediately above, the first slab. Required the perspective representation of the group of four slabs.

---

Upon the ground-line set off 6′ from **X** towards left, also 12′ from point 6 towards right, and join the points 6 12 to **C V**.

Again, set off 2′ from point 6 towards right; then, if point 2 be joined to **D 1**, the position of one corner of the base of the slab will be determined at **a**.

From **a** draw a line parallel to the ground-line to meet the line 12 **C V** at point **b**. *The line* **a b** *is the only visible edge of the base of the first slab.*

Imagine a vertical plane perpendicular to the picture to contain the left hand thickness of the slab. Its intersection with the ground-plane is represented by the chain-line 6 **C V**, and its intersection with the picture-plane is the vertical line (**I L** No. 1) drawn through point 6.

On **I L** (No. 1) set up $1\frac{1}{2}$′ from 6, and join point $1\frac{1}{2}$ to **C V**; then raise a perpendicular at **a** to meet the line previously drawn to **C V** at point **a′**.

Through point $1\frac{1}{2}$ draw a horizontal line to represent the **I L** of the upper surface of first slab. Upon this **I L** make the distance $1\frac{1}{2}$ o′ equal to 12′, and join point o′ to **C V**.

From **a′** draw the upper edge of the slab parallel to **a b**, to meet the line o′ **C V** at point **b′**.

Since the top of the slab is a square, and its near edge **a′ b′** is parallel to picture-plane, its diagonals must converge to **D 1**, **D 2**.

Join **a′** to **D 2**, also join **b′** to **D 1**; these lines intersect $1\frac{1}{2}$ **C V**, o′ **C V** at points **e f**, which are the back corners of the top of the slab Join **e f**.

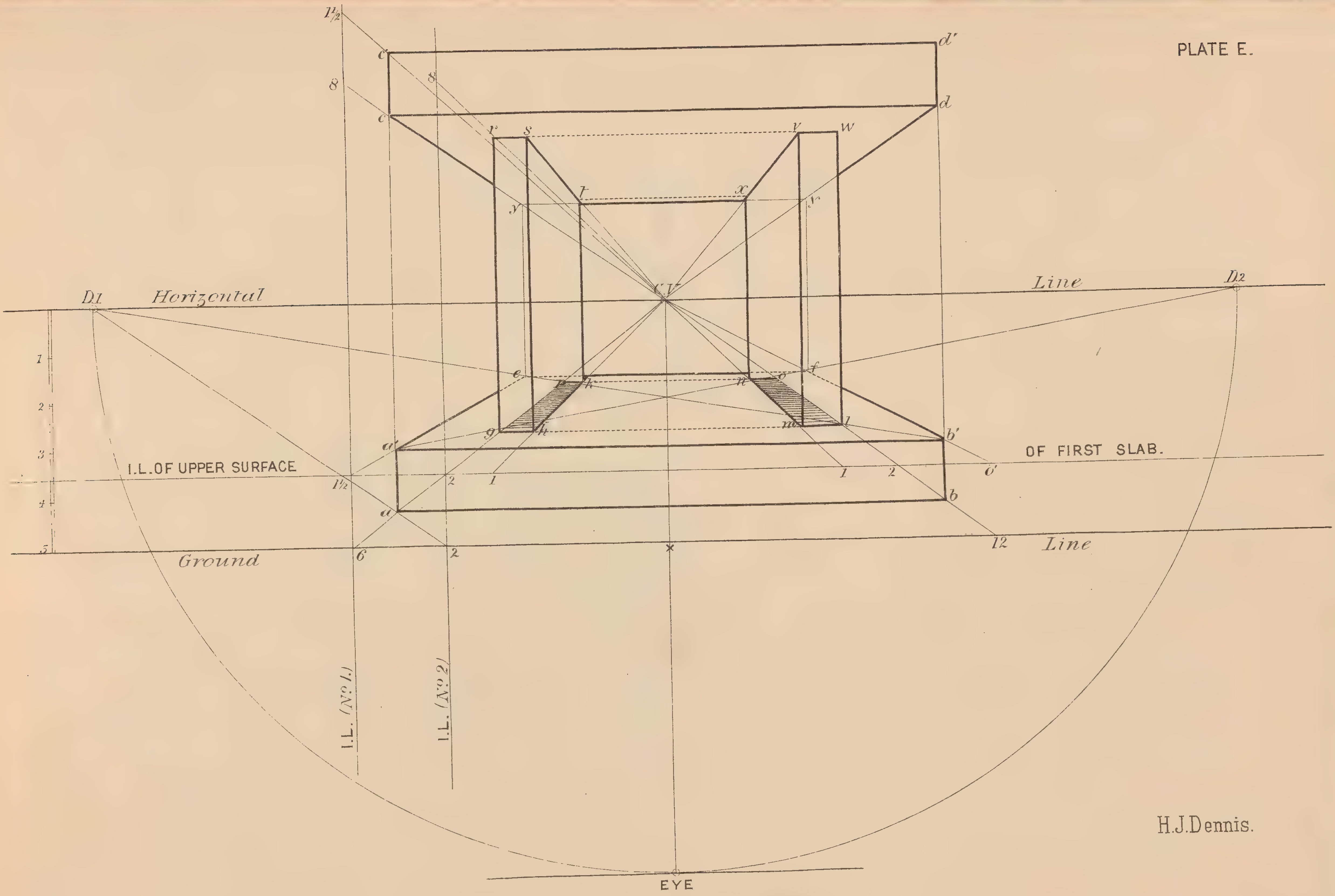
Upon the **I L** of upper surface of slab set off 2′ from $1\frac{1}{2}$ towards right, also 2′ from o′ towards left. Join the points 2 2 to **C V** giving the points **g l o p** on the diagonals, and if these points be joined, a square will result, which encloses the bases of the two smaller slabs.

The smaller slabs are to be 1′ thick; therefore, set off 1′ from the points 2 2, and join the points 1 1 to **C V**. These lines complete the bases of the slabs, **g h p k**, **l m o n**.

N.B.—The rectangular bases of the two slabs are filled in, that they may be better understood.

Find a vertical plane perpendicular to the picture-plane to contain the farther vertical face of the left-hand slab. Its intersection with the upper

PLATE E.

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surface of the first slab is shown by the chain-line **2 C V**, its intersection with the picture-plane is the vertical line **I L** (No. 2) drawn through point **2**.

Raise the perpendiculars at **g h k**, **l m n** : then set up **8′** from point **2** (*on* **I L** *of upper surface of slab*) on **I L** (No. 2). Join **8** to **C V**, which measures **g r** perspectively equal to **8′**.

A dotted line drawn from point **r**, parallel to **a b**, measures the vertical edges of the slabs **h s**, **m v**, **l w**.

Draw a line from **s** to **C V**, which meets the vertical line from **k** at **t**, and completes all that is visible of the left-hand slab.

Also draw a line from **v** to **C V**, which intersects the vertical line from **n** at point **x**, and completes the visible portion of the right-hand slab.

Upon **I L** (No. 1) set up two spaces, one equal to **8′**, and the other equal to $1\frac{1}{2}'$, for the thickness of the *fourth* slab; then join points **8**, $1\frac{1}{2}$ to **C V**.

Produce the line **a a′** upwards to meet **8 C V**, $1\frac{1}{2}$ **C V**, at points **c c′** also produce **b b** to meet the horizontal lines drawn from **c c′**, at points **d d′**.

Now join **c d** to **C V**, raise perpendiculars at **e f** to meet **c C V** **d C V** at points **y y**, and lastly join **y y** to complete the under surface of the fourth slab.

---

## EXERCISE.

1. A right cone stands on its base in the centre of the upper surface of the first slab. Diameter of base 4′, length of axis 6′.
2. In the centre, and tangential to the sides of each visible square face of the two smaller slabs, draw a circle.

## PLATE F.

Distance of eye in front of picture-plane, 12′.

Height of eye above the ground-plane, 5′.

Required the perspective representation of the group of four slabs, given in preceding exercise, when the square faces of the large slabs are vertical, and lie in planes perpendicular to the picture-plane. The outer square face of the left-hand large slab is 6′ on the left of the spectator, and the nearest corner is 1′ beyond the picture-plane.

---

Upon the ground-line set off 6′ from **X** towards left, and join point 6 to **C V**; also set off 11′ from 6 towards right, and join point 11 to **C V**.

*The outer surfaces of the large slabs will lie somewhere upon the lines* 6 **C V**. 11 **C V**.

Upon the ground-line set off 1½′ from 11 towards left, and 1½′ from towards right. Join the points 1½, 1½, to **C V**; upon these lines the inner surfaces of the large slabs will be determined.

Now obtain the nearest corner of the left-hand slab. Set off 1′ from 1½ towards right, and join 1 to **D** 1, giving **a** on the line 1½ **C V**.

Through point **a** draw a line parallel to the ground-line intersecting the lines 6 **C V**, 1½ **C V**, 11 **C V**, respectively, at the points **b c d**. Upon the ground-line set off 12′ from point 1 towards right; then draw a line from 12 to **D** 1, which intersects the line 1½ **C V** at point **e**, and determines the *inner* back corner of the left-hand slab.

Find an imaginary vertical plane to contain the inner square surface of the left-hand slab. Its intersection with the ground-plane is the chain-line joining 1½ to **C V**, and its intersection with the picture-plane is the vertical line (**I L**) drawn through point 1½.

Upon this **I L** set up 12′ from the ground-line, and join 12 to **C V**, then erect perpendiculars at **a e** to meet the line 12 **C V** at points **a′ e′**.

Now raise a perpendicular at **b**, and draw **a′ b′** parallel to **a b** for the thickness of the left-hand slab.

Draw **e f** parallel to **a c**; the line **c f** represents the lower *inner* edge of the second large slab.

Produce the upper edge **a′ b′** of the *first* slab, and to meet this line at **c′ d′**, draw verticals from **c d**. Complete the second slab by drawing a vertical line from **f** to meet the line **c′ C V**, at point **f′**.

### METHOD OF FINDING THE SMALLER SLABS.

Upon the ground-line set off 2′ from point 1 towards right, and 2′ from 12′ towards left; then join points 2 2 to **D** 1, meeting the chain-line 1½ **C V** at points **g h**, which are 2′ from the vertical edges of the left-hand slab.

From **g h** draw vertical lines upon the surface of the slab.

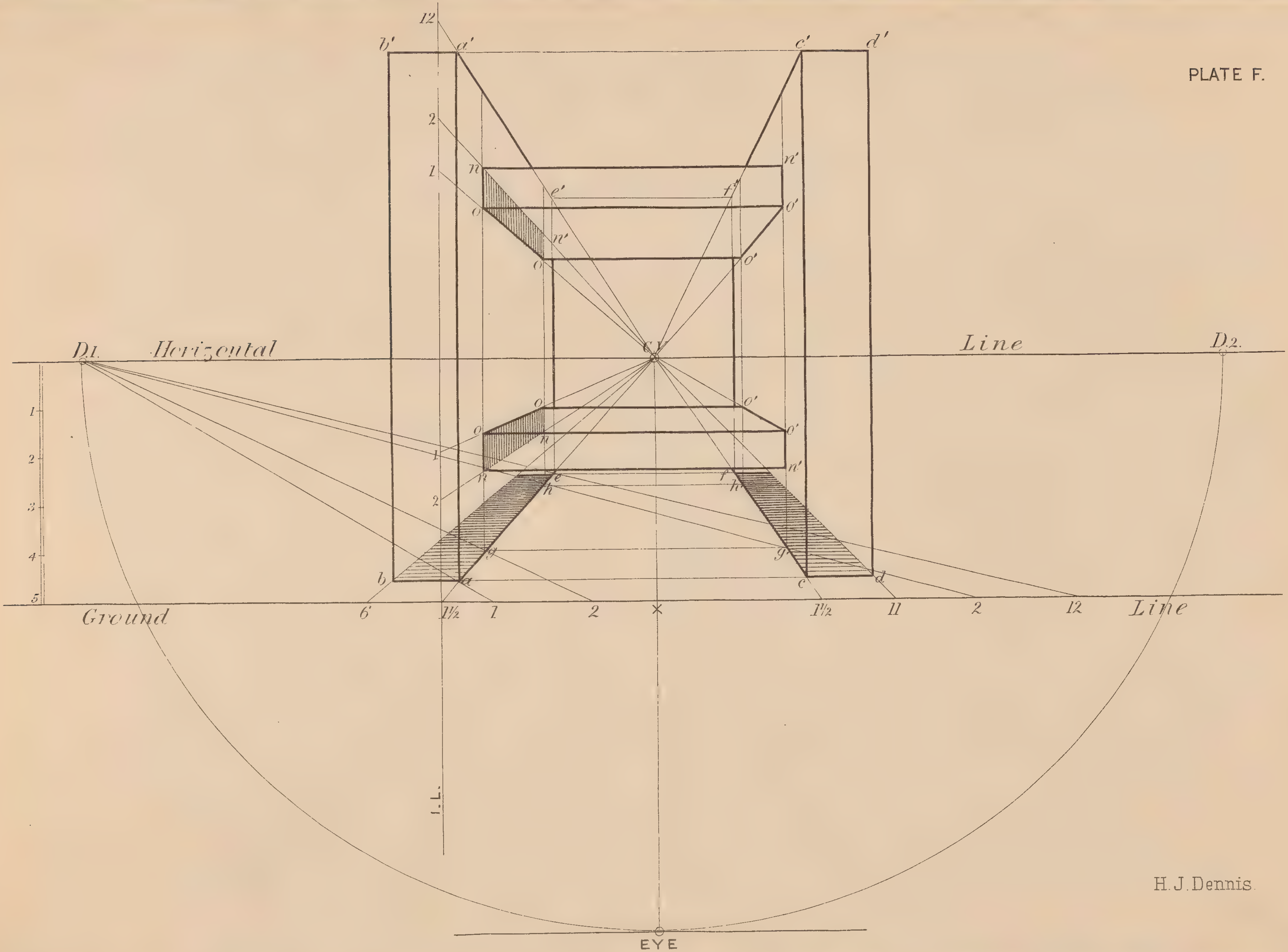
The lower surface of the lower slab and the upper surface of the upper slab are to be respectively 2′ from the horizontal edges of the left-hand slab; therefore, upon **I L** set up 2′ from 1½; also set down 2′ from 12; then draw lines from the points 2 2 to **C V**, which intersect the vertical lines drawn from **g h** at the points **n n n n**.

The slabs are to be 1′ thick; therefore upon **I L** set up 1′ from *lower* point 2; also 1′ below upper point 2; then join the points 1 1 to **C V**, giving the other four corners **O O O O** upon the verticals drawn from **g h**.

Now draw the lines **g g′**, **h h′**, parallel to **a c**, and upon the inner surface of the right-hand large slab draw vertical lines from the points **g′ h′**.

Lastly, draw the horizontal edges of the small slabs. **n n′**, **o o′**, parallel to **a c**, and join the points **o′ o′**, **o′ o′**, by lines converging to **C V**.

PLATE F.

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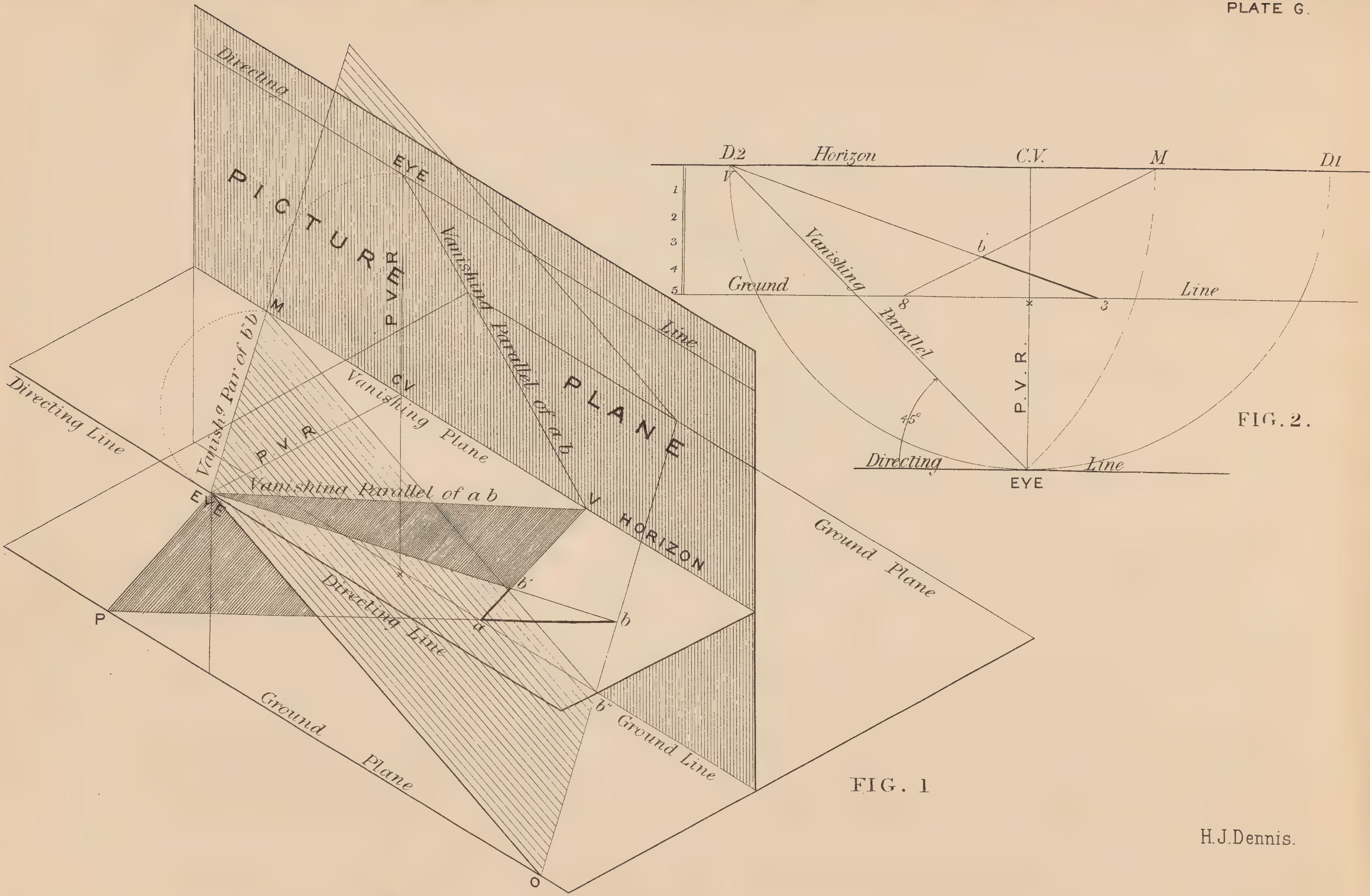


FIG. 1

FIG. 2.

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# PLATE G.

# ANGULAR PERSPECTIVE:

In the preceding plates the solids have their edges and surfaces *parallel* and *perpendicular* to the picture-plane. I now proceed to give a simple rule by which the vanishing point of any line may be obtained; also a few exercises which will enable the student to obtain the perspective representation of an object when its edges are inclined to the picture-plane at any angle, and lie in vertical or horizontal planes.

## RULE F.—TO FIND THE VANISHING POINT OF ANY LINE.

*Draw a line from the eye parallel to the original line* (the line whose vanishing point is required) *to meet the picture-plane.*

### FIG. 1.

Given the picture, ground and vanishing planes (*the latter plane is drawn through the eye parallel to the ground*); in these planes are also given the eye, **C V**, horizon, &c., to find the perspective representation of a line **a b**, lying upon the ground-plane behind, and receding from the picture-plane at an angle of 45° towards the spectator's right. The line **a b** is tangential to the picture-plane at point **a**.

### SOLUTION OF FIG. 1.

The vanishing parallel of **a b** must be first determined by Rule F. Draw from the eye a line making 45° towards right, with the *directing-line* to meet the picture-plane *upon the horizon* at point **V**, which is the vanishing point required.

The vanishing parallel of **a b** is really the intersection with the vanishing plane of a plane passing through the EYE and the line **a b** (see shaded plane EYE **V P a**).

NOTE.—Because **a b** lies on the ground plane, its vanishing plane is drawn through the EYE parallel to it, and appears to the spectator as a horizontal line, coinciding with the *horizon;* then, if the vanishing parallel of **a b** be drawn from the eye, in the vanishing plane, it will evidently appear as a simple point coinciding with **V** on the *horizon;* and if there were several lines behind the picture-plane parallel to **a b**, they would each converge to **V**, because there can be but *one* vanishing parallel to the whole.

The line **a b** touches the picture-plane at point **a**, which is the perspective representation of its nearer extremity **a**. It has been shown that **a b** will appear to converge to **V**; therefore join **a** to **V**; then draw a line from the farther extremity (**b**) of the given line to the eye, which represents a *ray of light* intersecting the picture-plane at point **b′**, and thus determining **a b′** upon the picture-plane as the representation of the given line **a b**.

In FIG. 1 the planes, lines, &c., are shown in their actual positions, but in *practice* everything must be imagined in the plane of the picture, as previously described; for example, the vanishing parallel is really in front of the picture-plane, but the student must imagine it revolved in the *vanishing plane*, upon **V** as centre, until it coincides with the *horizon* at **M** (*measuring point*) which actually represents the spectator's eye.

Again, the line representing the ray of light (b, EYE) is actually partly in front of, and partly behind the picture-plane; that portion *behind* is shown at b b′. and the remaining portion *in front* at b′ EYE.

We will now imagine a plane (EYE M O b″) to pass through EYE and the ray of light, this plane intersects the picture-plane in the line M b′ b″, and the ground-plane in the line O b″ b. *The line* M b″ b′ *is the vanishing line of this plane, the ray of light also vanishes in this line because it lies in the same plane.* It should be observed that this plane cuts the ground-plane in the line O b″ b, giving a b″ as the true length of a b on the ground line; and, that portion b″ b, *behind* the picture plane, forms the base of an *isosceles* triangle, of which a b, a b″, are the two equal sides.

Now, if we draw the vanishing parallel of b″ b from EYE to M, it is manifest that M is the vanishing point of b″ b, and if b″ be joined to M it will represent the line b″ b produced indefinitely in perspective; but, the ray of light drawn from b to EYE determines b″, and consequently a b′ is the perspective representation of a b, and, b″ b′ that of b″ b.

In all perspective drawings the vanishing plane (with the eye and vanishing parallel) is revolved into the plane of the picture upon the *horizon* as an axis either above or below. In the illustration FIG. 1, PLATE G, it is represented revolved upwards; and the EYE, *vanishing parallel*, *directing-line*, &c., are shown above the *horizon*; but, if we imagine the plane, &c., revolved below, the lines and point must be represented below the horizon.

NOTE.—If a line be drawn from the eye, *in the vanishing-plane*, to meet the picture-plane, it will make *equal angles* with the *Directing Line* and the *Horizon*; hence, in all our drawings, angles with the picture-plane must be calculated at the EYE with the *Directing Line.*

## FIG 2.

Distance of eye in front of the picture-plane, 12′.
Height of eye above the ground-plane, 5′.
Required the perspective representation of a right line lying upon the ground-plane, its nearer extremity touching the picture-plane at a point 3′ on the spectator's right. The line is 8′ long, and inclined at 45° to the picture-plane towards left.

## SOLUTION OF FIG. 2.

Prepare the paper for the perspective drawing by determining the *horizon, scale, ground-line,* C V, P V R, EYE, *directing-line*, &c.

Then set off 3′ from X, upon the ground-line, towards right, and point 3 will represent the nearer extremity of the required line in contact with the picture-plane. The vanishing parallel of the required line has to be next determined.

Draw a line from the eye, towards left, making an angle of 45° with the *directing-line* and meeting the horizon at point V (D 2).

NOTE.—This vanishing parallel gives the distance point as the vanishing point of *any* line inclined to the picture-plane at 45° towards left.

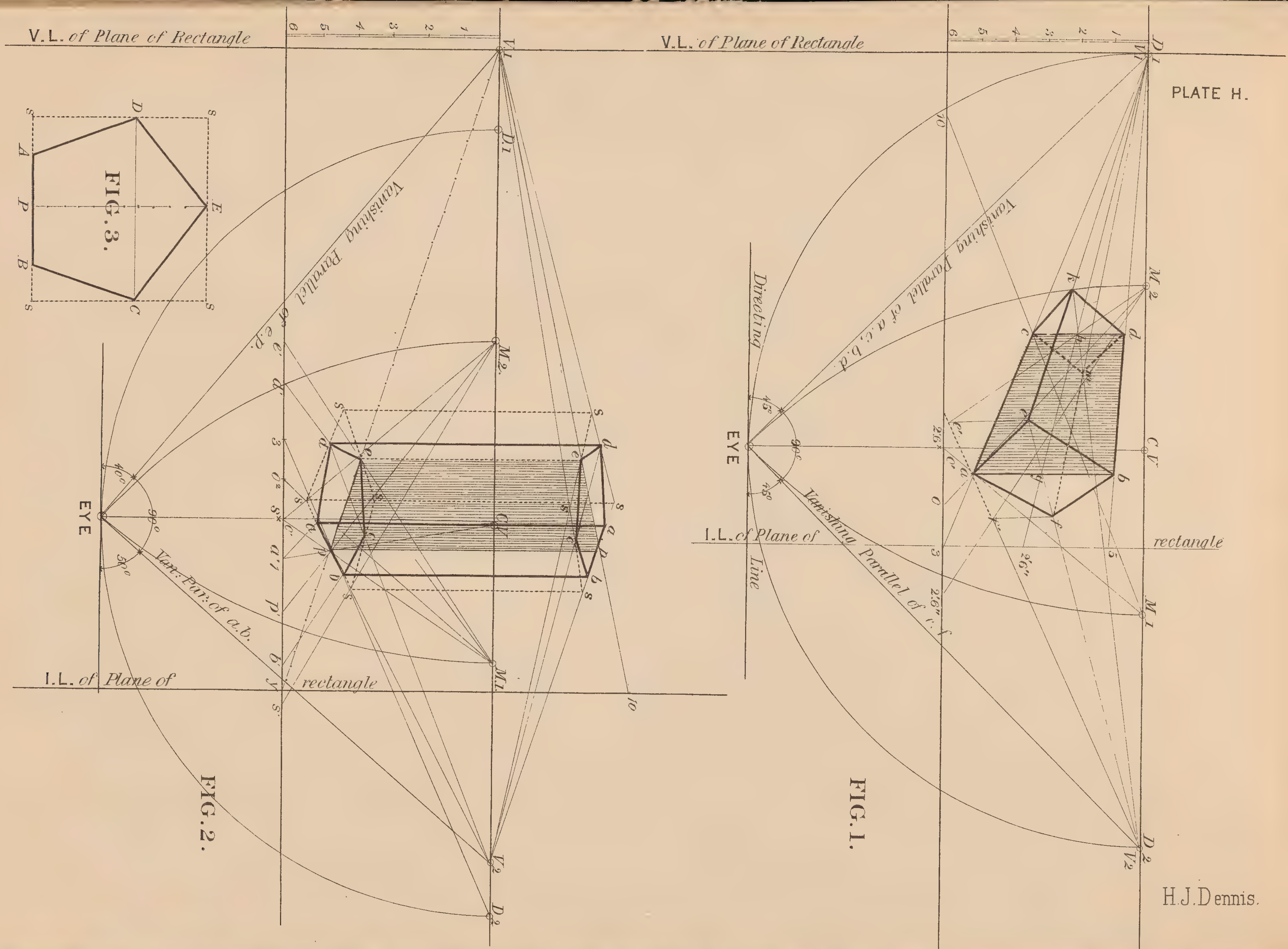
In order to find the measuring point of the required line, take point V as centre, with a radius equal to the length of the vanishing parallel, and describe an arc to meet the horizon at M. Point M represents the spectator's eye when brought into the surface of the picture-plane.

By referring to FIG. 1 it will be clearly seen that the required line must be measured by a straight line, representing a ray of light, drawn from its farther extremity to the eye; further, when this ray is brought into the surface of the picture-plane, its lower extremity is coincident with the ground-line at a distance from the nearer extremity of the given line equal to its actual length; and its upper extremity is at M.

The actual length of the required line is 8′; therefore upon the ground-line set off 8′ from 3 towards left, then join point 8 to M. The line 8 M is the representation of the ray of light in the plane of the picture.

Lastly, draw a line from 3 to V, where this line is intersected by the ray 8 M; the farther end of the required line is determined.

PLATE H.

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# PLATE H.

## FIG. 1.

Distance of eye in front of picture-plane, **12′**.

Height of eye above the ground-plane, **6′**.

I. Draw in perspective a *vertical* plane, receding from the picture-plane at an angle of **45°** towards left, its **I L** being **3′** on right of spectator.

II. A rectangle **10′** long, **5′** wide, lies in the above vertical plane, and stands upon the ground-plane on one of its long edges, the nearer extremity of which is **3′** from the **I L** of the vertical plane.

III. This rectangle represents the section of a right square prism made by a vertical plane passing through its axis, and two *opposite* long edges. Required the perspective representation of the prism.

---

*N.B.—A vertical plane inclined to the picture-plane intersects the ground-plane in a line; this line always makes the same angle with the picture-plane as the vertical plane.*

Having prepared the picture-plane, it is necessary to find the vanishing parallel of the intersection of the vertical plane with the ground-plane.

At the eye draw this vanishing parallel at **45°** with the *directing line* towards left, which gives **V 1** for the vanishing point of the intersection with the ground of the required vertical plane. (*See* Rule F and Fig. 1, Plate G).

*N.B.—Every vanishing point must have its own measuring point.*

*The vanishing and measuring points of a line must always be found on the* **V L** *of the plane in which the original line lies;* for example, *if a line lie upon the ground-plane, its* **V P** *and* **M P** *will lie upon the horizon, because this is the* **V L** *of the ground-plane.*

*The distance between a vanishing point and its measuring point must always equal the length of the vanishing parallel from such vanishing point.*

Find **M 1** by setting off the length of the vanishing parallel from **V 1** along the horizon.

Upon the ground-line set off **3′** from **X** towards right; through point **3** draw the **I L** of required vertical-plane; and the representation of its intersection with the ground-plane will be obtained by joining **3** to **V 1** by a chain-line. Through **V 1** draw the **V L** of the vertical plane.

In order to find the nearer extremity of the rectangle, set off **3′** from point **3** towards left (point **X**). Join point **X** to **M 1**, giving point **a** at **3′** from **I L** of vertical plane.

Now, set off **10′** from **X** towards left, and join point **10** to **M 1**, which intersects **3 V 1** at **c**, and gives **a c** as the lower edge of the rectangle upon the ground-plane.

Draw verticals at **a c**; and since they are contained by the vertical plane, it is obvious that they must be measured upon the vertical **I L** drawn through point **3**.

Set up **5′** from **3** and join point **5** to **V 1**; this line measures the vertical sides of the rectangle **a b, c d**. The rectangle is shaded that it may be more clearly understood.

In proceeding to obtain the nearer end of square prism, it should be observed that the edge of the rectangle **a b** is also one diagonal of the square, and the plane of the square is vertical and *perpendicular* to the vertical plane previously found.

*Find the vanishing parallel of the intersection with the ground-plane of a vertical plane containing the nearer end of prism.*

At the eye make a *right angle* with the vanishing parallel of **a c**, which gives **V 2** upon the horizon. Find **M 2** at a distance from **V 2**, equal to the length of the vanishing parallel of **e f**.

The *horizontal* diagonal of the square (*nearer end of prism*) must now be obtained; it will converge to **V 2**; but since it is above the ground-plane, the operation of measuring it will be much facilitated if we imagine a line lying upon the ground-plane immediately below it.

Through point **a** draw a line to **V 2**, and measure it perspectively equal to the diagonal of the square, one-half on left, the other on right of point **a**.

But point **a** is beyond the picture-plane; therefore, bring it forward to meet the ground-line at point **o**. On right and left of **o** set off 2′ 6″; then join the points 2′ 6″, 2′ 6″, to **M 2**, these lines intersect **a V 2** at points **e′** and **f′** which are immediately below the corners of the square required.

Upon **I L** of plane of rectangle set up 2′ 6″ from **3**. Join 2′ 6″ to **V 1**, giving points **g** and **h** as the centres of the ends of the prism.

Through point **g** draw a line to **V 2**, to meet this line at points **e** and **f**; raise perpendiculars at **e′** and **f′**.

Join **e b, b f, f a, a e,** for the nearer square end of prism.

Through point **h** draw a line to **V 2**; then join **f e** to **V 1**, meeting the horizontal diagonal of farther end of prism at points **m k**.

Complete the solid by joining **d k, k c, c m, m d**.

---

FIG. 2.

Distance of eye in front of the picture-plane, 11′.

Height of eye above the ground-plane, 6′.

I. A rectangle 10′ long, 5′ wide, stands upon the ground-plane on one of its short edges, and lies in a vertical plane which recedes from the picture-plane at an angle of 40° towards left. The nearer corner of the rectangle upon the ground-plane is 1′ on right of the spectator, and 3′ beyond the picture-plane.

II. Let this rectangle represent the section of a right pentagonal prism made by a vertical plane passing through its axis and one of its long edges.

*N.B.—The edge of the rectangle upon the ground-plane is the altitude of the pentagonal base of the prism.*

---

Upon the ground-line set off 1′ from **X** towards right, and join point **1** to **C V**; then set off 3′ from point **1**, and join **3** to **D 2**. These lines intersect and give the nearer corner of the rectangle upon the ground-plane (point **p**).

At the eye draw the vanishing parallel of **e p** at 40° with the directing-line towards left, to meet the *horizon* at **V 1**. Find **M 1**.

Through **V 1** draw a vertical line to represent to **V L** of plane of rectangle. Join **V 1** to **p** by a chain-line, and produce it to meet the picture-plane (on the ground-line) at point **y**. The chain-line **V 1, p y** is the intersection of the plane of rectangle with the ground-plane. Through **y** draw **I L** of plane of rectangle.

Upon the chain-line, from point **p**, find the lower edge of rectangle.

Bring forward **p**, by **M 1**, to meet the picture-plane at **O′**; then from **O′** set off **o′ e′** equal to 5′ *towards left*, and join **e′** to **M 1**, thus giving point **e** on the chain-line as the farther corner of the rectangle.

Now draw perpendiculars at **p e**; set off **y 10** on **I L** of plane of rectangle equal to 10′, and join point **10** to **V 1**. This line determines the perspective height of the rectangle, also its upper edge **e p**.

The plan of the base of the prism should be next obtained (*See* FIG. 3).

Draw a line **E P** equal in length to the width of the rectangle, viz. 5′. Upon **E P** as an *altitude* construct a regular pentagon **A B C D E**; then enclose the pentagon within a rectangle **S S S S**.

By referring to FIG. 3, the student will observe that the side of the pentagon **A B**, passes through point **P** at right angles to the altitude **E P**. The line **e p** vanishes at 40° with the picture-plane towards left; and if a right angle be made at the eye, with the vanishing parallel of **e p**, the vanishing parallel of **a b** will be found; and its vanishing point is **V 2**. Find **M 2**.

We will now proceed to find the representation of the rectangle, which encloses the pentagonal base of the prism.

Through **p** draw a line to **V 2**; then bring forward **p** by **M 2** to meet the ground-line at **p′**; on right and left of **p′** set off **p′ s′**, **p′ s′**, equal to **P S**, FIG. 3. Join **s′ s′** to **M 2**, giving two corners of the circumscribed rectangle at **s s**, upon the line drawn from **p** to **V 2**.

Join the points **s s**, last obtained, to **V 1**, and the back corners **s s** are determined upon these lines, by producing a line through point **e** and **V 2**.

Draw the sides of the rectangle at the top of the prism through **e** and **p** to **V 2**, and determine the lengths of these lines by raising perpendiculars from the points **s s s s** upon the ground-plane.

Having completed the imaginary rectangular prism, which encloses the required pentagonal prism, we have simply to find the corners of the pentagonal ends upon the sides of the rectangles.

Upon the ground-line set off **s′ a′**, **s′ b′**, respectively equal to **S A**, **S B**, FIG. 3; then join **a′ b′** to **M 2**, these lines intersect the nearest side of rectangle at **a b**, and if these points be joined, an edge of the pentagon will be obtained.

Bring forward the nearest corner of the rectangle (point **s**) by **M 1** to meet the ground-line at $O^2$; then set off $O^2$ **d′** towards left equal to **S D**, FIG. 3. Join **d′** to **M 1**, which gives **d** another corner of the pentagonal base upon the ground-plane.

In FIG. 3 the line **D C** is parallel to **A B**; therefore, if a line be drawn from **d** to **V 2**, corner **C** will be found at the point of intersection upon the opposite side of the circumscribed rectangle.

Join **a d**, **d e**, **e c**, **c b**.

The long edges of the pentagonal prism are obtained by drawing vertical lines from points **a b c d e** upon the ground to meet the edges of the upper circumscribed rectangle at corresponding points **a b c**, &c.

Complete the solid by joining the upper points **a b**, **b c**, **c e**, **e d**, **d a**.

## EXERCISE.

Distance of eye in front of the picture plane, 12′.
Height of eye above the ground plane, 5′. Scale, ½″ to 1′.

A right square prism has a base of 4′ side, and is 7′ high. This prism stands on the ground plane on its square base, the sides of which recede from the picture plane at angles of 40° and 50° respectively on the right and left of the spectator, and its axis is 1′ on right and 6′ beyond the picture plane.

# PLATE I.

Distance of eye in front of the picture-plane, 12′.

Height of eye above the ground-plane, 5′.

I. A square plane of 10′ edge lies flat upon the ground-plane; two of its edges recede from the picture-plane at an angle of 50° towards right, and its nearest corner is 2′ on spectator's left and 2′ beyond the picture-plane.

II. Find the centre of the right edge of square plane, and let it represent the nearest corner of a second square plane (whose plane is coincident with that of first square), the edges of which are 4′ long, and make angles of 10° with those of the former square plane.

---

Upon the ground-line set off 2′ on left of **X**, and join point **2** to **C V**; also set off 2′ from point **2** towards right, and join point **X** to **D 2**. The intersection of these lines is the nearest corner of the first square plane.

Draw the vanishing parallel of the near side of square plane at the eye, making an angle of 50° with the directing line towards right, and meeting the horizon (**V L** *of plane of square*) at **V 1**. Find **M 1**.

Join **a** to **V 1**; then bring forward point **a** by **M 1** to meet the ground-line at point **o**. Set off 10′ from **o** towards right, and join point **10** to **M 1**, the nearest edge, **a b**, of the square plane is thus determined.

The adjacent edges of the square are actually at right angles to **a b**; therefore at the eye, with the vanishing parallel of **a b**, draw their vanishing parallel at 90°; or, if 40° be made with the directing line towards left, the same result will be obtained, viz., *a right angle between the two vanishing parallels at the eye.*

Having found **V 2**, its measuring point, **M 2**, must be determined; then join **a** to **V 2** and measure the length of the edge of the square (*perspectively*) upon it by **M 2**.

Bring forward **a** by **M 2** to meet the ground-line at point **o′**. Set off 10′ from **o′** towards left, and join **10** to **M 2**. This line gives corner **c** of the square.

Draw **c d** to **V 1**, because it is actually parallel to **a b**; also draw **b d** to **V 2**.

**a b c d** is the representation of the first square plane.

Next determine the centre of the edge **a b**.

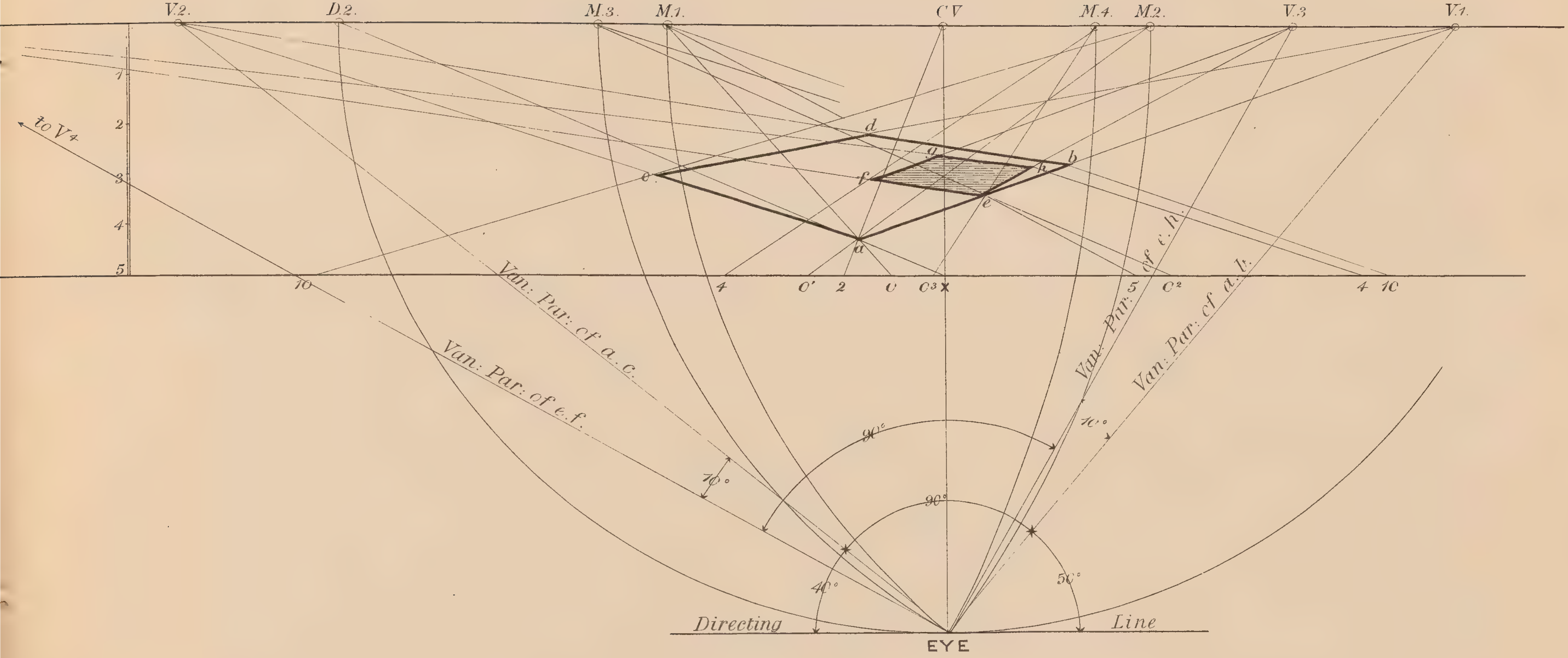
Upon the ground-line set off **O 5**, equal to half the actual length of edge of square; then join **5** to **M 1**, giving (**e**) the centre of **a b**.

## METHOD OF FINDING THE VANISHING PARALLELS OF THE SECOND SQUARE PLANE.

*NOTE.—In order to find the representation of two original lines making any given angle with each other, it is necessary to draw their vanishing parallels at the eye, making the angle between them equal to the actual angle between the original lines.*

Find the vanishing parallel of **e h**, by drawing a line from the eye at

PLATE I.

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10° with the vanishing parallel of a b; then, if a line be drawn from the eye at 90° with the vanishing parallel of e h; or at an angle of 10° with the vanishing parallel of a c, *towards left*, the vanishing parallel of e f will be obtained.

Points V 3, V 4, upon the horizon are the vanishing points of the edges of the *second* square plane. Find M 3, M 4.

Join e to V 3, also draw a line from e to V 4.

Bring forward e by M 4 to meet the ground-line at point $O^3$. Set off 4′ from $O^3$ towards left, and join point 4 to M 4; this line intersects e V 4 at f, and gives one edge of required square.

Again, bring forward e by M 3, to meet the ground-line at point $O^2$. Set off 4′ from $O^2$ towards right, and join point 4 to M 3, giving another corner of the square at point h.

Lastly, draw h g to V 4, and f g to V 3.

---

## EXERCISE TO BE WORKED ON PLATE I BY THE STUDENT.

Upon the larger square a b c d, construct a short right prism 2′ high. The smaller square e f g h is the *plan* of a right pyramid, 6′ high, which stands upon the upper surface of the prism.

# PLATE J.

"The centre of vision is given. The eye is 12′ *by scale,* distant from it and 5′ above the ground-plane.

"Represent the cube and circular disc shown *half size* in the accompanying plan and elevation. The solids rest upon the ground-plane, and the diagonal **A B** is horizontal and recedes towards the right hand at 50°. The angle **A** touches the picture-plane at a point vertically below the centre of vision."

*N.B.—The above is a copy of an examination paper set by the Department of Science and Art in May,* 1876.

---

Upon the **P V R**, set off **X A** equal to twice the distance **A E** in elevation, then find the vanishing parallel of the horizontal diameter, **A B**, by drawing a line from the eye at an angle of 50° with the directing-line, towards right hand, meeting the *horizon* at **V 1**.

Since the horizontal diameter, **A B**, vanishes to **V 1**, and the face of the cube to which it belongs is vertical, it is evident the plane of that face (**A B C D**) must vanish in a vertical line drawn through **V 1**.

Draw the **V L** of plane **A B C D** through **V 1**, and find **M 1**; also show its intersection with the ground-plane by drawing a chain-line from **X** to **V 1**.

Upon the ground-line set off **X c′**, **c′ b′**, equal to twice the distances **E C**, **C E**, in the elevation, then join **c′ b′**, to **M 1**. These lines intersect the chain-line **X V 1** at points **C b**, which lie upon the ground-plane immediately below the corners **D B** of the cube. Point **C** is the representation of the nearer extremity of the edge upon which the cube rests; it is also the lowest corner of the face **A B C D**.

Through **X** draw **1 L** of plane **A B C D**, and upon it set off **X d**, equal in length to twice the given diagonal **C D** in the elevation.

Draw a vertical line from **C**, then draw a line from **d** to **V 1**, giving upper corner **D** of the cube upon the former line.

Draw a line from **A** to **V 1** and raise a perpendicular at **b** to meet it at **B**. Join **A C**, **C B**, **B D**, **D A**, which completes the nearest face of the cube.

The *horizontal* edges of the cube are actually at right angles to the plane of **A B C D**; therefore, to find their vanishing point it is necessary to make a right angle at the eye with the vanishing parallel of **A B**. **V 2** is their vanishing point. Find **M 2**.

Join **A**, **B**, **C**, **D**, to **V 2**, then find a line lying upon the ground-plane immediately below the horizontal edge of cube **A A**.

Join **X** to **V 2**. Upon the ground-line set off **X e′**, **e′ e′**, **e′ a′**, respectively equal to twice the lengths of **A E**, **E E**, **E A**, in plan; then by joining **e′**, **e′**, **a′** to **M 2**, the corresponding distances in perspective will be determined upon the line drawn from **X** to **V 2**.

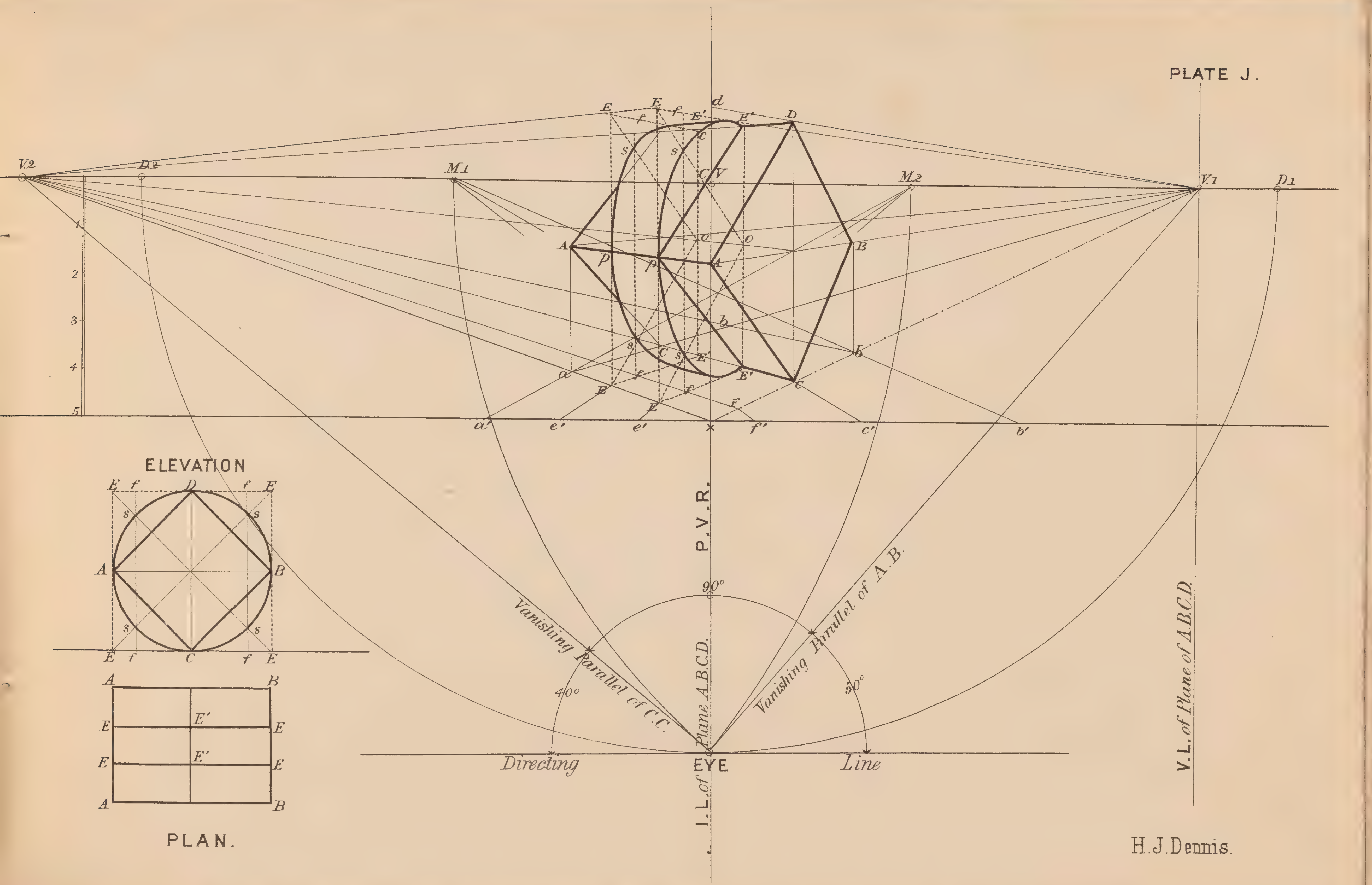
Join **A** to **V 2**, then raise a perpendicular at point **a**, which determines **A A**, the horizontal edge of the cube.

Now join **a** to **V 1**, also join **b** to **V 2**; these lines intersect at **b**. which is a point upon the ground-plane immediately below the farther extremity of the horizontal edge, **B B**, of the cube.

Draw a line from **C** to **V 2**, to meet **a b** at **C**; also draw a line from **D** to **V 2** to meet a vertical line drawn from the farther extremity, **C**. which determines the upper corner of the back face of cube.

Join **A C**, **A D**, for the visible portion of the cube.

# PLATE J.

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## METHOD OF FINDING THE DISC.

Describe a square about the elevation of the circular disc, draw the diameters **A B**, **C D**, and the diagonals **E E**, **E E**, which intersect the circle at points **s**, **s**, **s**, **s**; through these points draw **f f**, **f f**, parallel to the sides of the square.

*N.B.—The invisible portion of the disc is intentionally omitted, that the work may be more intelligible to the student.*

The points **E**, **E**, previously found upon the line **X a**, are the positions of the nearest corners, upon the ground, of the squares which circumscribe the front and back faces of the disc.

Since only one half of the disc is to be found, it is necessary to find but one-half of the circumscribed squares.

Make the lines **E E′**, **E E′** converge to **V 1** and stop upon the lower edge of the cube. The lines **E E′**, **E E′**, represent the semi-bases of the squares.

At the points **E′**, **E′**, upon **C C**, draw verticals to intersect the upper edge of the cube at **E′**, **E′**. The lines **E′ E′**, **E′ E′**, are the vertical diameters of the disc and circumscribed squares.

The axis of the disc lies upon the axis of the cube, and is found by drawing a line from the centre of the near face of cube to **V 2**, which intersects the vertical diameters of the disc (**E′ E′**) at points **O**, **O**.

The near vertical edges of the squares must be now determined. Draw vertical lines from the points **E E** (upon **X a**), their perspective heights are found by drawing a line from **d** (on **1 L** of **A B C D**) to **V 2**. Join the upper points **E E′**, **E E′**.

Draw the semi-diagonals, **o E**, **o E**, of the two squares, then upon the ground-line set off **X f′** equal to twice the length of **E f** in elevation. Join **f′** to **M 1**, which intersects **X b** at **F**.

Join **F** to **V 2**, which intersects the semi-bases of the squares at **f**, **f**, and from these points draw verticals to meet the semi-diagonals at points **s**, **s**; **s**, **s**.

Through the points **E′**, **s**, **p**, **s**, **E′**, in each semi-square, draw the curved line representing the visible portion of the disc.

Lastly, draw tangents to these curves to **V 2**, for the thickness of the disc.

---

### EXERCISE.

Distance of eye in front of the picture plane, 12′.
Height of eye above the ground plane, 5′. Scale, ½″ to 1′.

1. A right cylinder lies upon the ground plane on its side; its line of contact with that plane recedes from the picture plane towards right, at an angle of 40°. The near extremity of the line of contact with the ground is 2′ on right, and 3′ beyond the picture plane. Length of cylinder 7′, diameter of its ends 4′.

2. A right square prism stands vertically on the ground plane on its base, behind the cylinder and touching it at a point midway between its circular ends. Two rectangular faces of the prism make 40° towards right with the axis of the cylinder. The prism is 4′ 6″ square and 8′ long. Represent the two solids in perspective.

### EXERCISE (Art Exam., 1892).

Distance of eye in front of picture plane, 12′.
Height of eye above the ground plane, 5′.

Two equal circles, each 6′ in diameter, intersect, so that their planes are at right angles to each other, and their centres are at the same point. One of these circles is in a vertical plane, which recedes from the picture plane at the angle of 40° towards the right, and the circumference touches the ground plane at a point 1′ on left, and 3′ from the ground line. The plane of the other circle is horizontal. Represent these two circles in perspective.

# PLATE K.

## FIG. 1.

Distance of eye in front of the picture-plane, **12′**.

Height of eye above the ground-plane, **5′**.

A regular pentagon, **8′** edge, has its surface on the ground-plane, and one corner coincident with the picture-plane at **2′** on the spectator's right. The edges of the pentagon which meet at a point on the picture-plane recede from it at **36°** on right and left of the line of direction.

*The vanishing points of the pentagon must be obtained.*

## METHOD OF FINDING THE ACTUAL ANGLES OF THE EDGES OF THE PENTAGON WITH THE PICTURE-PLANE.

Construct a regular pentagon upon a line **8′** long; then through corner **A** draw a line representing the *plan* of the picture-plane at an angle of **36°** with the edges **A B**, **A D** of the pentagon.

By producing **B C** to meet the plan of the picture-plane at point **1**, its actual angle will be found equal to **A 1 B** towards left, because **B C** is on the left of the dotted line **1 2**, which is *perpendicular* to the picture-plane.

---

Upon the ground-line set off **x a** equal to **2′**. Point **a** is the nearest corner of the pentagon.

From the eye draw the vanishing parallels of **A B**, **A D** at **36°** with the directing-line, and produce them to meet the horizon at **V 1**, **V 2**. Find **M 1**, **M 2**.

Join **a** to **V 1** and **V 2**; set off **a b′**, **a d′**, respectively equal to **A B**, **A D** in plan; then join **d′** to **M 2**, and **b′** to **M 1**; these lines determine the perspective lengths of **a d**, **a b**.

Now draw the vanishing parallel of **B C** at the same angle with the directing line *towards left*, as the original line **B C** makes with the plan of the picture.

By referring to the plan the student will observe that the edge of the pentagon **D E** makes the same angle with the plan of the picture as the edge **B C**, but in the *opposite* direction.

*N.B.—The student may easily determine if a line incline to right or left, by making a perpendicular to the picture-plane; then, if the original is on left of the perpendicular, it will vanish to the left hand.*

Draw the vanishing parallel of **D E** at the same angle with the directing line as that of **B C**, but on the opposite side of the eye.

Having determined **V 3**, **V 4**, find **M 3**, **M 4**.

Bring forward **b** by **M 4** to meet the ground line at point **O**; make **O c′** equal to **B C** in plan, and join **c′** to **M 4**. This line intersects **b V 4** at **c**; consequently, **b c** is the representation of another edge of the required pentagon.

Next, bring forward **d** by **M 3** to meet the picture-plane at **o′**; set off **o′ e′** equal to **D E** in plan, and draw a line from **e′**, which intersects **d V 3** at **e**, and gives another corner of the pentagon.

The remaining edge of the pentagon is *parallel* to the picture-plane, and found by joining **e c**.

## FIG. 2.

Distance of eye in front of the picture-plane, **12′**.

Height of eye above the ground-plane, **5′**.

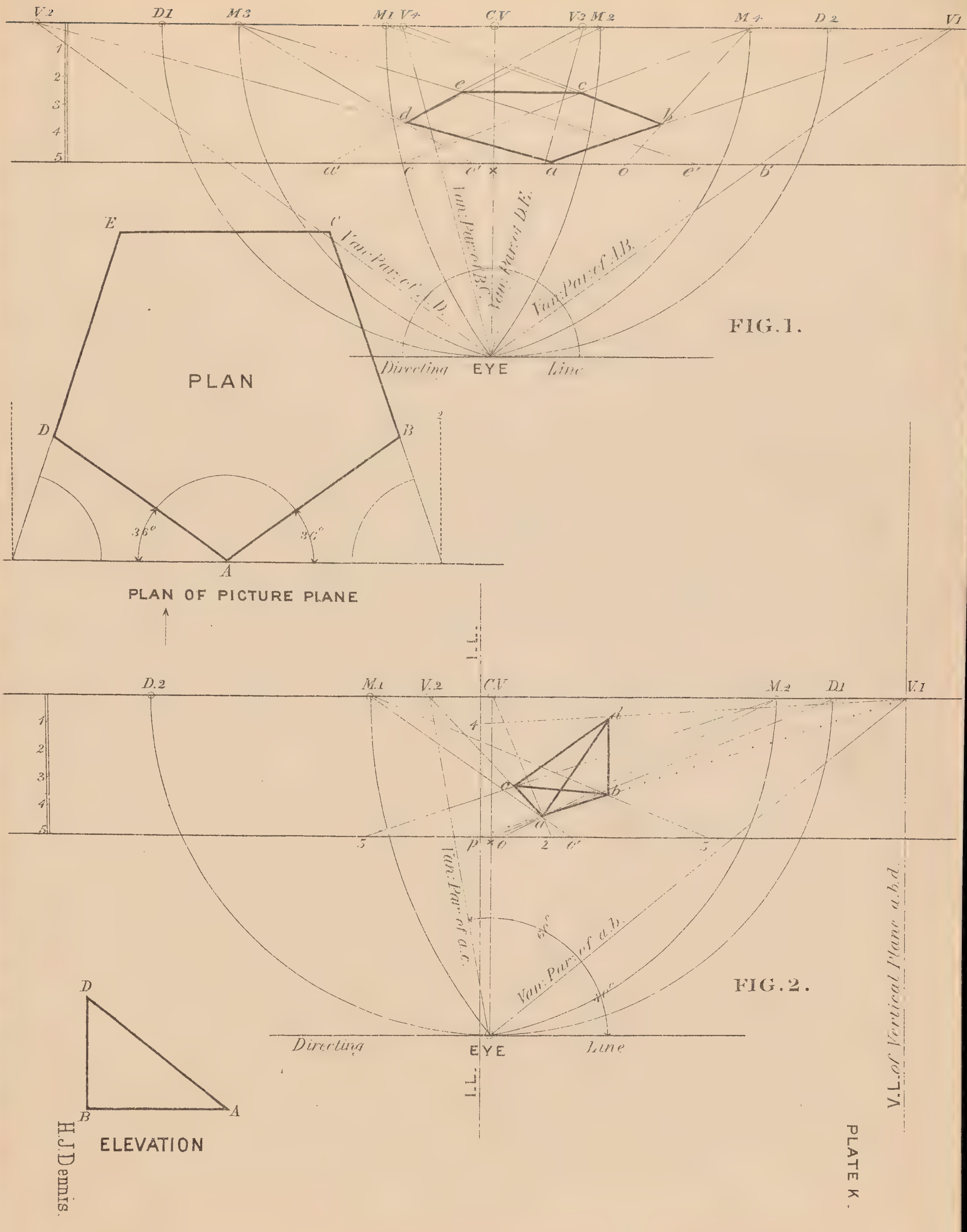


PLATE K.

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I. An equilateral triangle, 5′ edge, lies upon the ground-plane; one of its edges recedes from the picture-plane at an angle of 40° towards right. Nearest corner 2′ on right of line of direction, and 2′ beyond the picture-plane.

II. At the right-hand corner of the triangle draw a vertical line 4′ long

III. Join the upper extremity of the vertical line to the other corners of the triangle, and give the actual length of the lines, also their actual angle with the ground.

---

## SOLUTION OF FIG. 2.

Upon the ground-line set off 2′ from **X** towards right, and join point **2** to **C V**; then, by joining **X** to **D 1**, the near corner of the triangle will be found at point **a**.

Draw the vanishing parallel of one side of the triangle (**a b**) at 40° with the directing line towards right, which gives **V 1** upon the horizon. Find **M 1**.

Join **a** to **V 1**; then bring forward **a** by **M 1** to meet the ground-line at point **O′**. Make **O′ 5** equal to 5′, and join point **5** to **M 1**; the right corner of the triangle is thus determined at point **b**.

The actual angle between the adjacent sides of an equilateral triangle is 60°; therefore at the eye make the vanishing parallel of **a c** at 60° with the vanishing parallel of **a b** *towards left*, which determines **V 2** upon the horizon. Find **M 2**.

Join **a** to **V 2**; then bring forward **a** by **M 2** to meet the ground-line at **O**; set off 5′ from **O** towards left, and join point **5** to **M 2**; this line gives the remaining corner (**c**) of the triangle. Join **c b**.

At **b** erect a perpendicular. In order to measure this line, the student must conceive it to lie in a vertical plane with the edge **a b** of the triangle.

The intersection of the vertical plane with the ground-plane is shown by the chain-line drawn through **a b** to **V 1**; its **V L** is the vertical line drawn through **V 1**, and its **I L** is drawn vertically through point **p**.

Upon this **I L** set up 4′ from **p**; join point **4** to **V 1**, giving **d** as the upper extremity of the required vertical line. Join **d a**, **d c**.

Before proceeding to find the actual length and angle of the line **a d**, the student should carefully observe that **a b d** is a right-angled triangle, and **a d** is its *hypothenuse.*

We already know the actual lengths of the base (line **a b**) and perpendicular (**b d**); therefore make *an elevation* of the right angle formed by these two lines. **A B** equal to 5′ and **B D** perpendicular to it, 4′ long. Now, to find the length of the hypothenuse, we have simply to join **A D**, and its actual angle with the ground is **D A B**.

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## GENERAL INSTRUCTIONS FOR FINDING THE ACTUAL LENGTH AND ANGLE WITH THE GROUND OF ANY INCLINED LINE.

*I. Always imagine the inclined line to be the hypothenuse of a right-angled triangle.*

*II. The inclined line must be contained by a vertical plane, and its intersection with the ground-plane determined.*

*III. From the upper extremity of the inclined line let fall a perpendicular to meet the intersection of vertical plane with the ground, which determines the length of the base of the right-angled triangle.*

*IV. Ascertain the lengths of the base and perpendicular, and place them at right angles to each other; then join their extremities, which will determine the actual length and angle required*

## PLATE L.

Distance of eye in front of the picture-plane, 12′.

Height of eye above the ground-plane, 5′.

I. Represent the letter **A** shown *half-size* in the accompanying plan and elevation. The letter stands vertically upon the ground-plane, its nearest corner being 2′ on spectator's right and 2′ beyond the picture-plane, and its parallel vertical faces lie in planes which recede from the plane of the picture at 40° towards left hand.

II. A point **P** is given upon the ground-plane, which is to be joined to the angle **P′** of the letter **A**. You are required to give the *actual* length and angle with the ground-plane of the inclined line **P P′**.

*N.B.—A portion of the above is taken from the examination paper, May,* 1877.

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Upon the ground-line set off 2′ from **X** towards right, and join point 2 to **C V**. Now join **X** to **D 1**, which determines the nearest corner (a) upon the ground-plane.

At the eye draw the vanishing parallel of 40° with the picture towards left, giving **V 1** upon the horizon. Find **M 1**.

Through a draw a chain-line to **V 1**, and produce it to intersect the ground-line at point **A**. This chain-line is the representation of the intersection of the vertical plane of nearer face of letter with the ground-plane. Its **V L** is drawn through point **V 1**, and its **I L** through point **A**.

The lower edge of the letter must be represented upon the chain-line, because it is the intersection of the ground and vertical plane of face of letter.

Bring forward a by **M 1** to meet the ground line at point a′; make the distances a′ b′, b′ c′, c′ d′, d′ e′, e′ f′, f′ g′, respectively equal to *twice* a b, b c, c d, d e, e f, f g; next join the points a′, b′, c′, &c., on the ground-line to **M 1**, giving the representation of the corresponding points a, b, c, &c., on the chain-line.

Upon **I L** of face of letter set up **A** t′, t′ s′, s′ o′, o′ m′, respectively equal to twice the corresponding distances in the elevation, and join t′ s′ o′ m′ to **V 1**.

Draw vertical lines from e c (*on chain-line*) to meet m′ **V 1** at m″ n″. The line m″ n″ is the upper edge of the nearer face of letter.

Join m″ a, n″ g, for the inclined edges of the letter. Next draw a vertical line from point d to meet O′ **V 1** at O″. Join o″ f, o″ b, and where these lines intersect t′ **V 1**, s′ **V 1**, the horizontal bar of the letter will be obtained.

Draw the vanishing parallel of the thickness of the letter from the eye at right angles to the former vanishing parallel, or at 50° towards right, with the directing line.

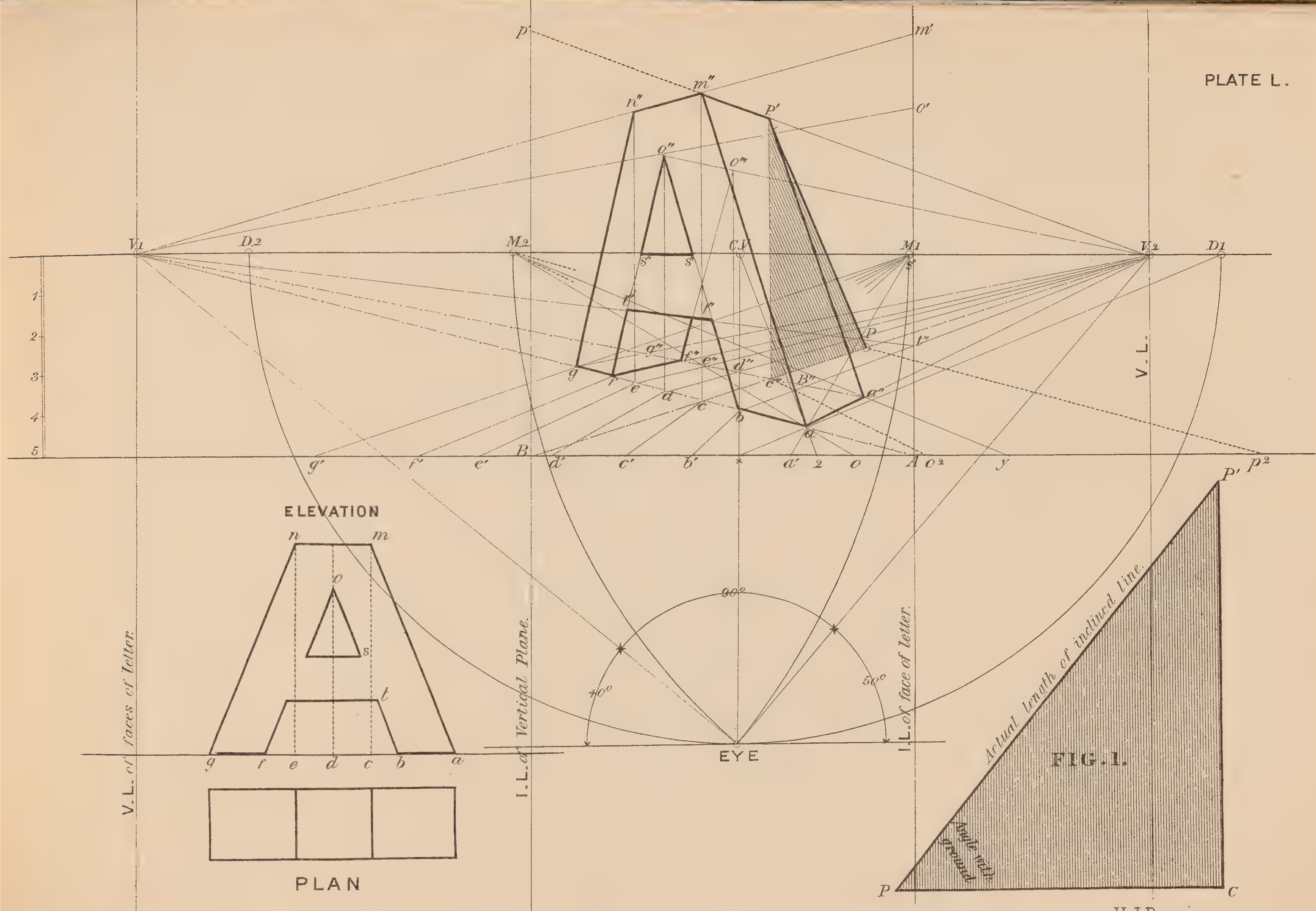
**V 2** is the vanishing point of all the lines forming the thickness of the letter. Determine **M 2**.

Bring forward a by **M 2** to meet the ground-line at point o; make o y equal to *twice* the breadth of the plan, and join y to **M 2**, giving point a″ on the line drawn from a to **V 2**.

From the points a, b, c, &c., on the lower edge of front surface of letter draw lines to **V 2**; also draw a line from a″ to **V 1**, where the former lines are intersected by the latter, points b″, c″, d″, &c. are found upon the lower edge of farther face of letter.

Join m″ to **V 2** and raise a perpendicular to meet it at **P′**; also

PLATE L.

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raise a perpendicular at **d″** to meet a line drawn from **o″** to **V 2** at point **o‴**.

Join **f″ o‴ f f″**; and from **s″** draw a line to **V 2**; this line completes all that is visible of the letter.

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Draw the inclined line **P P′**.

*N.B.—In order to determine the length of an inclined line we must either be able to find a point on the ground under its upper extremity, or we must know at what angle its vertical plane recedes from the picture-plane.*

In the present instance the point on the ground immediately below **P′** is already determined at **c″**.

Join the points on the ground (**c″ P**) by a chain-line, and produce it to meet the horizon at **V 2**; also to meet the ground-line at **B**.

*A right-angled triangle is formed by the lines* **P P′, P′ c″, c″ P**.

Through **V 2** draw **V L** of vertical plane, and through **B** draw its **I L**.

Next determine the actual length of **c″ P** by **M 2**. Bring forward points **c″ P** by **M 2** to meet the ground-line at $o^2$, $p^2$, and the distance between these points is equal to the actual length of the *base* of the right-angled triangle.

Make **C P**, Fig. 1, equal to $o^2$ $p^2$.

The actual height of the perpendicular is determined thus:—Bring forward **P′** by **V 2** to meet **I L** of vertical plane at **p′**.

Make **C P′**, Fig. 1 equal to **B p′**, at right angles to **C P**; then join **P P′**, Fig. 1, which gives the actual length of the *hypothenuse* of the right-angled triangle (inclined line).

The actual angle of the inclined line with the ground is shown between the base (**C P**) and the hypothenuse (**P′ P**) Fig. 1. (Angle **C P P′**).

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EXERCISE.

Distance of eye in front of the picture plane, 12′.

Height of eye above the ground plane, 5′. Scale, ½″ to 1′.

1. A prism is 8′ long, and its ends are hexagons of 1′ 6″ side. This prism rests on the ground plane on one of its rectangular faces, the long edges of which are inclined to the picture towards right at 40°, and the nearest corner of the prism upon the ground plane is 2′ on left of the spectator, and 2′ beyond the picture plane.

2. A circle of 5′ diameter stands upon the upper rectangular face of the hexagonal prism, and lies in a vertical plane parallel to, and midway between, the ends of the prism. Represent the prism and circle in perspective.

Distance of eye, 12′.

Height of eye, 5′.

Find a point **A**, 3′ on left, 3′ above the ground plane and 2′ from the picture plane. Find a second point **B**, 4′ on right, 3′ above the ground and 8 from the picture plane. Join these points by a right line **AB**, and from the extremity **A** cut off a distance representing 3′.

# EXAMINATION EXERCISES.

*N.B.—The problems marked with an asterisk (*) are copied from examination papers which have been set by the Science and Art Department, South Kensington. In every case the student should use a scale of half an inch to the foot, and the lines used in working the problems must be shown.*

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November, 1877.

Ex. 1.

* The centre of vision is given. The eye of the spectator is to be 12′, by scale, distant from it, and 5′ above the ground-plane.

I. Two square right prisms of the same form and dimensions stand upon the ground-plane; the axes and two faces of each prism are in vertical planes receding from the picture-plane at 35° towards left. The prisms are each 6′ high, 2′ square at the base, and 6′ apart. The nearest point of the nearer solid, upon the ground-plane, is 1′ on the spectator's left, and 3′ from the picture-plane.

II. Resting upon these two, and projecting at 1′ at either end, is a third prism, its base being a square of 2′ side, its length 12′, and its axis lies in the same vertical plane as those of the other prisms. Give a perspective representation of the three solids.

*One hour and a half allowed.*

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November, 1877.

Ex. 2.

Distance of eye in front of the picture-plane, 12′.

Height of eye above the ground-plane, 5′.

* I. An equilateral triangle of 10′ side, lies upon the ground-plane, its nearest angle being 2′ on the spectator's left and 2′ from the picture-line. One side of the triangle vanishes towards left hand at an angle of 45° with the picture-plane.

II. This equilateral triangle is the lower face of a solid slab 2′ in thickness.

III. Two similar slabs, having same thickness, but with sides respectively of 8′ and 6′, are placed upon the first so as to form three steps. The centres of these three solids lie in the same vertical line, and their horizontal edges recede to the same vanishing points on the right and left of the spectator. Give a perspective representation of the solids. (*See Plate* **P.**)

*One hour and a half allowed.*

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November, 1877.

Ex. 3.

Distance of eye in front of the picture-plane, 12′.

Height of eye above the ground-plane, 5′.

* I. A square slab of 8′ side, and having a thickness of 1′, lies upon the ground-plane, its vertical faces receding from the picture-plane at angles of 45°. The point upon the ground-plane which is nearest to the spectator is 2′ on the left of centre of vision, and 1′ from the picture line.

II. A right truncated cone stands upon its smaller end, in the centre of the upper face of this slab. The axis of the truncated cone is 6′ in length, and the diameters of its plane circular surfaces are 6′ and 4′ respectively. Give a perspective representation of the two solids.

*One hour and a half allowed.*

May, 1876.

Ex. 4.

* The centre of vision is given. The eye of the spectator is to be 12′, by scale, distant from it, and 5′ above the ground-plane.

I. A cube of 5′ stands upon the ground-plane on one of its faces; its horizontal edges make 40° and 50° with the picture-plane. The nearest point on the ground-plane is 3′ from the picture-line and 2′ on right of spectator.

II. A square slab of 8′ side and 2′ thickness rests upon the upper face of the cube, its edges projecting equally on all sides, and its vertical faces being parallel to the vertical faces of the cube. Give a perspective representation of the two solids.

*One hour allowed.*

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May, 1876.

Ex. 5.

* The centre of vision is given. The eye of the spectator is 12′, by scale, distant from it, and 5′ above the ground-plane.

I. A right prism, 9′ long, and having for its ends a rectangle of 1′ × 1½′, lies upon the ground-plane on one of the wider of its long faces. Nearest corner on the ground-plane 3′ from picture-line and 2′ to right. Long edges of solid vanish to left hand at 45° with the picture-plane.

II. The solid is the lowest of a flight of three steps. Represent the flight of steps in perspective.

III. Upon the top of third step place a semicircular arch, external span of which is 9′, internal span is 6′, and the thickness is the same as the breadth of the step.

*One hour allowed.*

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Ex. 6.

Distance of eye in front of the picture-plane, 16′.
Height of eye above the ground-plane, 7′.

Three circular slabs rest upon each other and form *in plan* three concentric circles of 4′, 8′ and 12′ diameters respectively. The largest slab being at the base, and the smallest one uppermost. The thickness of each slab is 1½′. You are required to draw these solids when the large slab rests upon the ground-plane on its thickness; its plane circular surfaces lie in vertical planes which recede from the picture-plane at 35° towards left, and the nearest point in its circumference is 3′ on spectator's right and 5′ beyond the picture-plane. *The smallest slab is nearest to the picture-plane.*

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Ex. 7.

Distance of eye in front of the picture-plane, 14′.
Height of eye above the ground-plane, 8′.

I. A square prism 10′ long, 4′ edge of base, lies upon the ground-plane upon one of its rectangular faces, its long edges being parallel to the picture-plane, and the nearest corner upon the ground-plane is 3′ on right and 7′ from the picture line.

II. A hexagonal prism 12′ long, 3′ edge of base, rests on the upper surface of the square prism on one of its rectangular faces, and the centres of these faces of the prisms are coincident. The long edges of the hexagonal prism make an angle of 45° with the axis of the square prism, towards right. (*See Plate* Q.)

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Ex. 8.

Distance of eye in front of the picture-plane, 14′.
Height of eye above the ground-plane, 5′.

I. Two rectangular planes, 8′ × 5′, stand vertically upon the ground-plane on their short edges, and meet each other at an angle of 45°. The plane nearer to the spectator recedes from the picture-plane at an angle of 80° towards right, and its nearer corner upon the ground-plane is 2′ on the spectator's left and 5′ beyond the picture-plane. *The second plane vanishes to the left, and the two planes meet at the farther vertical edge of the nearer rectangular plane.*

II. Required two other vertical planes, of the same dimensions as preceding, in such a position that they will trisect the angle contained by the first two planes.

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Ex. 9.

Distance of eye in front of the picture-plane, 14′.

Height of eye above the ground-plane, 7′.

I. A slab, 8′ square and 1′ thick, rests upon the ground-plane on one of its square faces; two of its vertical sides recede from the picture-plane at equal angles. Near corner of slab 3′ on spectator's left and 1′ beyond the picture-plane.

II. Upon the upper surface of square slab rests a solid having a square base, the corners of which coincide with the centres of the upper edges of square slab. The upper surface of the second solid is also a square, having its sides parallel to those of base of second solid, and its diagonal is 3′ 4″ long. The centre of the top is immediately over the centre of the base and 10′ from it. Required the perspective representation of the two solids.

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Ex. 10.

* The centre of vision is given. The eye of the spectator is to be 12′, by scale, distant from it, and 6′ above the ground-plane.

I. A hexagonal pyramid, sides of base 2′, height 7′, stands upon the ground-plane; the centre of the base is 4′ from the picture line and 1′ on the spectator's left. Two edges of the base are parallel to the picture-plane.

II. A square slab, 1′ thick, is pierced through its centre by this pyramid, and rests in a horizontal position upon it at a level of 2′ from the ground-plane. Its long edges are at equal angles with the picture-plane, and its nearest corner touches it. (*See Plate* **R.**)

*One hour allowed.*

Ex. 11.

Distance of eye in front of the picture-plane, 14′.

Height of eye above the ground-plane, 6′.

I. A square of 9′ edge, has its surface upon the ground-plane, its contiguous sides make equal angles with the picture-plane, and its nearest corner is 3′ on left of the line of direction and 1′ beyond the picture-plane.

II. Subdivide the square into 9 other equal squares.

III. The centre square is the base of a right pyramid, 7′ high, and the squares occupying the corners of the original square are the bases of cubes. Complete the representation of the solids.

IV. Join the left hand corner of the large square to the apex of the pyramid; give the actual length of the line, and its angle with the picture and ground-planes.

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Ex. 12.

Distance of eye in front of the picture-plane, 14′.

Height of eye above the ground-plane, 5′.

I. Find a point (**A**) on the picture line 4′ on left of spectator. From **A** draw a line receding from the picture at 60° towards right. On this line find two points, **B** and **C** respectively, equal to 3′ and 6′ from **A**. Let the line **B C** be the side of an equilateral triangle, upon which, as a base, rests a right pyramid 6′ high.

II. Draw in perspective, upon the ground-plane, a circle which will circumscribe the base of the pyramid.

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Ex. 13.

Distance of eye in front of the picture-plane, 12′.

Height of eye above the ground-plane, 5′.

I. Find a point (**A**) lying on the ground-plane, 3′ on left of line of direction, and 2′ beyond the picture-plane.

II. Draw four lines, each starting from point **A**, 5′ long, and lying on the ground-plane.

1st. Parallel to picture, towards right.
2nd. Perpendicular to picture.
3rd. Receding from picture at 30° towards left.
4th. Receding at 20° with 2nd line, towards right.

III. Join the extremities of the lines so as to form an *irregular* polygon.

IV. You are required to draw, upon any convenient part of your paper, a geometrical plan of the polygon, and mark the length of each line forming its sides.

V. At point **A** draw a vertical line **3′ 6″** high, join its upper extremity to the corners of the irregular polygon, and give the actual length, angle with the picture, and angle with the ground-plane, of each inclined line.

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Ex. 14.

Distance of eye in front of the picture-plane, **12′**.

Height of eye above the ground-plane, **5′**.

I. Draw in perspective a pyramid, its base being an equilateral triangle of **5′** edge, and height **7′**. Its nearest corner is **3′** on the spectator's left, and one side of the plan recedes from the picture-plane at an angle of 65° towards right.

II. Upon the apex of the pyramid is balanced a circular plate of iron; its plane is horizontal, and its circumference touches the picture-plane.

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Ex. 15.

I. Put into perspective a hexagonal slab, **4′** edge of base, and **3′** thick. This slab rests upon the ground-plane on its base; its axis is **3′** on left of **C V**, and **5′** beyond the picture-plane. Two contiguous edges of the base make equal angles with the picture-plane.

II. Upon the upper surface of the slab rests a square pyramid, **6′** high, and the *diagonal* of its base is **6′**. The centre of the base of the pyramid coincides with the centre of the upper surface of slab. One diagonal of base of pyramid makes an angle of 40° with the picture-plane towards right. Required its perspective representation when the eye is **14** in front of the picture-plane and **5′** above the ground.

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Ex. 16.

Distance of eye in front of the picture-plane, **9′**.

Height of eye above the ground-plane, **3′ 6″**. *Scale* **$1\frac{3}{4}''$** *to* **1′**.

An iron pail is, *in elevation*, a frustum of a cone supported by a cylindrical rim **3″** deep. The diameters of the top, lower conical portion, and cylindrical rim at base, are respectively **2′ 6″**, **1′ 9″**, and **1′ 9″**, and the total height is **2′ 3″**. Required its perspective representation when its axis is **2′** beyond the picture and **2′** on right of spectator. The handle, which is semicircular, equal in diameter to the top of the pail, lies in a vertical plane, which recedes from the picture-plane at an angle of 40° towards left.

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Ex. 17.

Distance of eye in front of the picture-plane, **14′**.

Height of eye above the ground-plane, **6′**.

I. A hexagonal pyramid, **3′** edge of base, **8′** high, rests upon the ground-plane on its base, its axis is **1′** on the left of spectator and **6′** beyond the picture-plane. One diagonal of the base is inclined to right, at an angle of 80° with the picture-plane.

II. An equilateral triangular slab, **1′ 6″** thick, is pierced through its centre by the pyramid, and rests in a horizontal position at a level of **2′** from the ground-plane. The *corners* of the *slab* and the *alternate corners* of the *base* of the *pyramid* lie in the *same vertical planes*, and one corner of the slab touches the picture-plane.

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Ex. 18.

Distance of eye in front of the picture-plane, **12′**.

Height of eye above the ground-plane, **5′**.

I. A right cylinder, 10′ long, 4′ diameter, rests on the ground-plane on its side; the line of contact with the ground recedes from the picture-plane at an angle of 40° towards left, the near extremity of which is 3′ on right of line of direction, and 3′ beyond the picture-plane.

II. Leaning against the cylinder is a rod, 12′ long, lying in a vertical plane *perpendicular* to the axis of the cylinder, midway between its extremities. The lower end of the rod is upon the ground-plane, and 6′ from the line of contact of cylinder. Required the actual angle of the rod with the ground-plane, also the imaginary section of the cylinder made by the vertical plane of the rod. *The point of contact of the rod with the cylinder must be shown.*

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Ex. 19.

Distance of eye in front of the picture-plane, 14′.

Height of eye above the ground-plane, 5′.

I. Required the perspective representation of a circle, 7′ diameter, lying in a horizontal plane 5′ *above* the eye, the nearest point in the circumference being 4′ on the left of the spectator and 2′ beyond the picture-plane.

II. Upon the ground place a circle, 7′ diameter, immediately below the other. Let these circles represent the ends of a cylinder; complete it.

III. Find a point (**A**) on the ground-line at 2′ on left of spectator. Point **A** is one extremity of a rod which lies in a vertical plane with the axis of the cylinder. You are required to show the point of contact of the rod with the curved line of upper end of cylinder, and, the imaginary section of the cylinder made by the plane of the rod.

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Ex. 20.

Distance of eye in front of the picture-plane, 16′.

Height of eye above the ground-plane, 6′.

I. Two contiguous walls of a room meet at right angles in a vertical line 12′ high, 14′ beyond the picture, and 2′ on the left of the spectator The wall which is on right of the **C V** recedes from the picture-plane at an angle of 40° towards left.

II. The floor is composed of rectangular tiles, the sides of which are parallel to the lower edges of the walls, and, respectively 4′ and 6′ long.

III. In the right hand wall draw an opening 7′ high, 3′ 6″ wide, to represent a doorway. The door opens into the room having its hinge-joint on the farther edge of doorway. Draw the door opened at 60° with the surface of the right hand wall and vanishing towards right. The hinge-joint is 6′ from the corner of the room.

IV. In the left hand wall draw an opening 6′ high, 3′ broad, its farther vertical edge being 4′ from the corner of the room, and its horizontal edges are, respectively, 3′ from floor and ceiling lines. This opening represents the position of a window, show how to divide it into nine equal rectangles to represent the sash for the window panes.

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Ex. 21.

Distance of eye in front of the picture-plane, 12′.

Height of eye above the ground-plane, 6′.

Scale, $\frac{1}{2}$″ to 1′.

I. *Five slabs, each 1′ thick, are to be shown, placed as though they formed a rectangular box without a lid, standing on the ground-plane, the outside measurements of which are, length 8′, breadth 6′, and height 3′; near angle, 7′ on left and 4′ beyond the picture-plane. The long edges vanish to right at 40° with the picture-plane.

II. A line **A B**, 6′ long, lies upon the ground-plane, perpendicular to the picture-plane, having its nearer extremity 5′ on the right of spectator, and 4′ beyond the picture-plane. Let this line be a diameter of a circle lying on the ground-plane.

III. The circle is to be the base of a right cylinder 7′ high. Complete it.

*One hour and a half allowed.*

May, 1878.

Ex. 22.

Distance of eye, 12′.

Height of eye, 5′.

Scale, $\frac{1}{2}''$ to 1′.

I. *A regular octagon of $2\frac{1}{2}'$ side lies upon the ground-plane, with two sides perpendicular to the picture-plane. The angle nearest the spectator is 2′ on his right, and 2′ from the picture-line.

II. This octagon is the base of a right prism $1\frac{1}{2}'$ in length.

III. Another prism of the same form and dimensions stands upon the ground-plane on one of its rectangular faces, its octagonal surfaces lying in vertical planes receding at 45° to the right. One edge of the prism touches the picture-plane at 5′ on the left of the spectator.

*One hour and a half allowed.*

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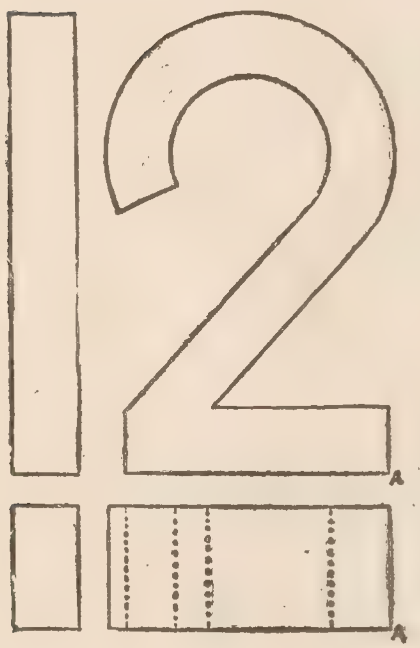
November, 1878.

Ex. 23.




Distance of eye in front of the picture-plane, 12′.

Height of eye above the ground-plane, 5′.

Scale $\frac{1}{2}''$ to 1′.

* Give the perspective representation of the two figures shown by plan and elevation, *half size,* in the accompanying diagram. The point **A** touches the ground-plane at 4′ on right of the spectator and 2′ from the picture-line, and the faces of the figures lie in vertical planes vanishing to left at angles of 45° with the picture-plane.

*One hour and a half allowed.*

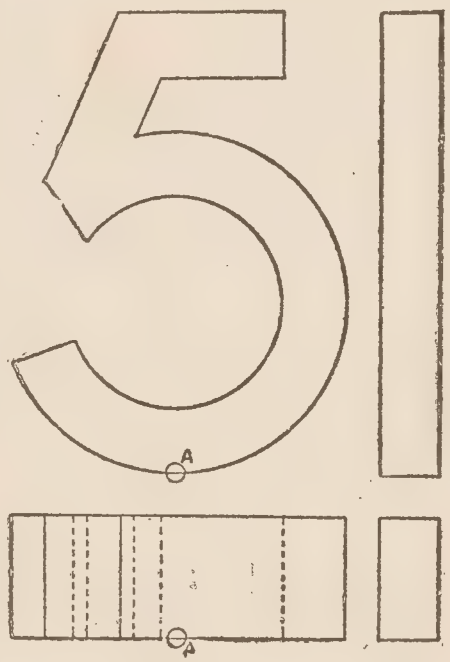
November, 1878.

Ex. 24.

Distance of eye in front of the picture-plane, 12′.

Height of eye above the ground-plane, 5′.

Scale, $\frac{1}{2}''$ to 1′.




* Give the perspective representation of the two figures shown by plan and elevation, *half size,* in the accompanying diagram. The point **A** touches the ground-plane 1′ on the left of spectator, and 4′ from the picture-line, and the faces of the figures lie in vertical planes vanishing to the right at angles of 40° with the picture-plane.

*One hour and a half allowed.*

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May, 1879.

Ex. 25.

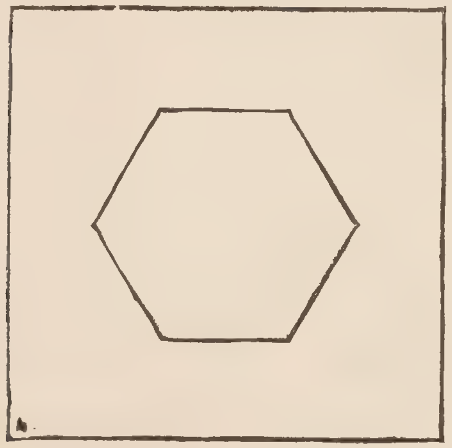
Distance of eye in front of the picture-plane, 12′.

Height of eye above the ground-plane, 5′.

Scale, $\frac{1}{2}''$ to 1′.

* Give the perspective representation of the square right prism and the hexagonal right prism shown by end elevation, *half size,* in the ac-

companying diagram. The square prism is 3′ in length and is penetrated



by the hexagonal prism, which latter projects $1\frac{1}{2}'$ from each of the two square faces. The point **A** on the nearer square face of the prism is to be upon the ground-plane, 2′ on right of the spectator and 2′ beyond the picture-plane. The axes of the solids are to vanish to the left at an angle of 55° with the picture-plane.

*One hour and a half allowed.*

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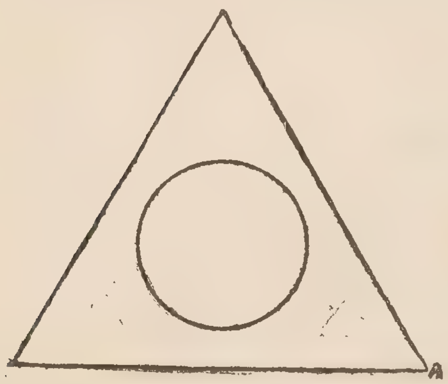
May, 1879.

Ex. 26.

Distance of eye in front of the picture-plane, 12′.
Height of eye above the ground-plane, 5′.
Scale, $\frac{1}{2}''$ to 1′.




* Give the perspective representation of the triangular right prism and the cylinder shown by end elevation, *half size*, in the accompanying diagram. The prism is 3′ in length, and is penetrated by the cylinder, which latter projects $1\frac{1}{2}'$ from each of the triangular faces of the prism. The point **A** on the nearer face of the prism is to be on the ground-plane, 1′ on the right of the spectator, and 2′ from the picture-line. The axes of the solids are to vanish towards the right at an angle of 50° with the picture-plane.

*One hour and a half allowed.*

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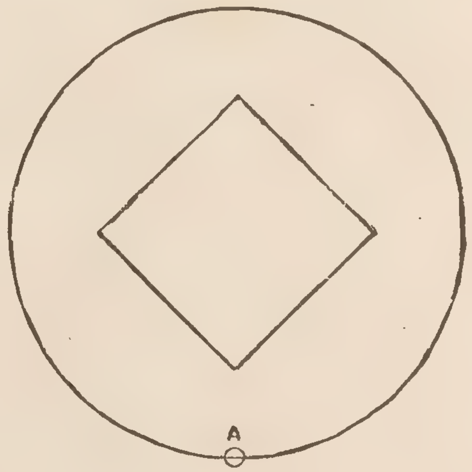
May, 1879.

Ex. 27.

Distance of eye in front of the picture-plane, 12′.
Height of eye above the ground-plane, 5′.
Scale, $\frac{1}{2}''$ to 1′.

* Give the perspective representation of the right cylinder and right prism shown by end elevation, *half size*, in the accompanying diagram. The cylinder is 3′ in length and is penetrated by the prism, which latter




projects $1\frac{1}{2}'$ from each face of the cylinder. The point **A** on the nearest face of the cylinder is to be on the ground-plane, 1′ on the spectator's right hand and 5′ from the picture-line. The axes of the solids are to vanish at 50° towards left with the picture-plane.

*One hour and a half allowed.*

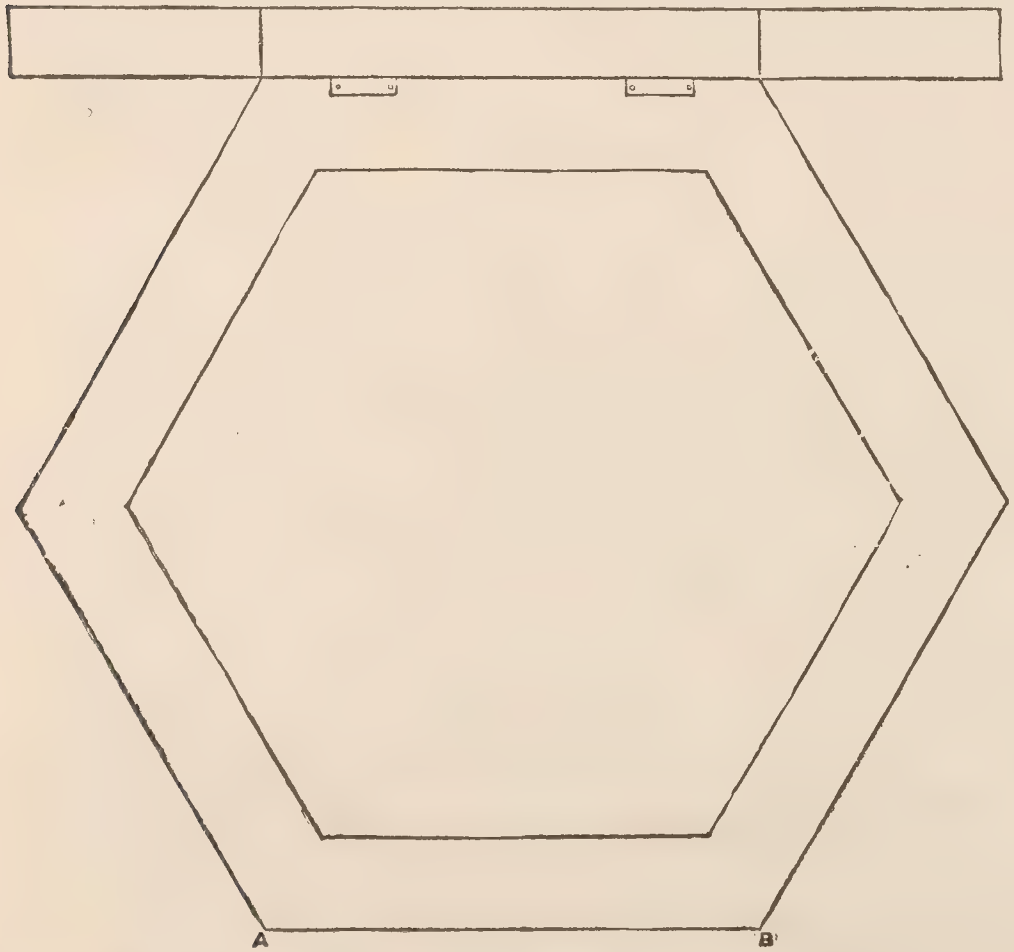
November, 1879.

Ex. 28.

* The centre of vision is given, the eye of the spectator is to be **12′**, by scale, distant from it, and **5′** above the ground-plane. Scale, ½ inch to 1 foot.

The lines used in working the problems *must* be shown.

The given figure is the plan of a hexagonal box and lid. The box




stands upon the ground-plane, and the lid is turned upwards on its hinges into a vertical position. The point **A** (on the ground-plane) is **1′** on the spectator's left hand, and **2′** from the picture-line; and the line **A B** recedes at an angle of **40°** towards right. The height of the box (exclusive of the lid) is **2′, 6″**. Give the perspective representation, showing only the visible lines.

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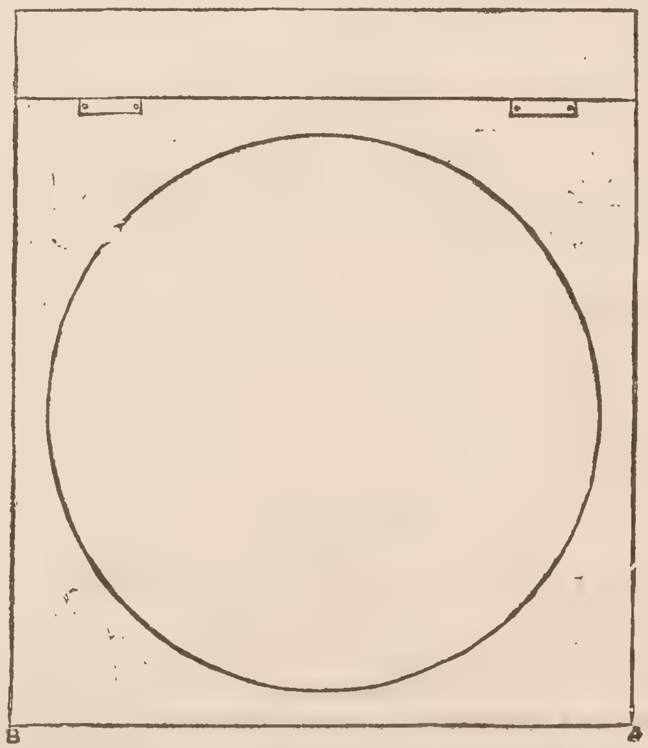
November, 1879.

Ex. 29.

* The centre of vision is given, the eye of the spectator is to be **12′**, by scale, distant from it, and **5′** above the ground-plane. Scale, ½ inch to **1** foot.

The lines used in working the problem *must* be shown.




The given figure is the plan of a square box and lid. The box stands upon the ground-plane, and is cylindrical inside; the lid is turned

upward on its hinges into a vertical position. The point A (on the ground-plane) is **1′** on the spectator's left hand and **1′** from the picture-line, and the line **B A** recedes towards the left at an angle of **35°**. The height of the box (exclusive of the lid) is **2′, 6″**. Give the perspective representation, showing only the visible lines.

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November, 1879.

Ex. 30.

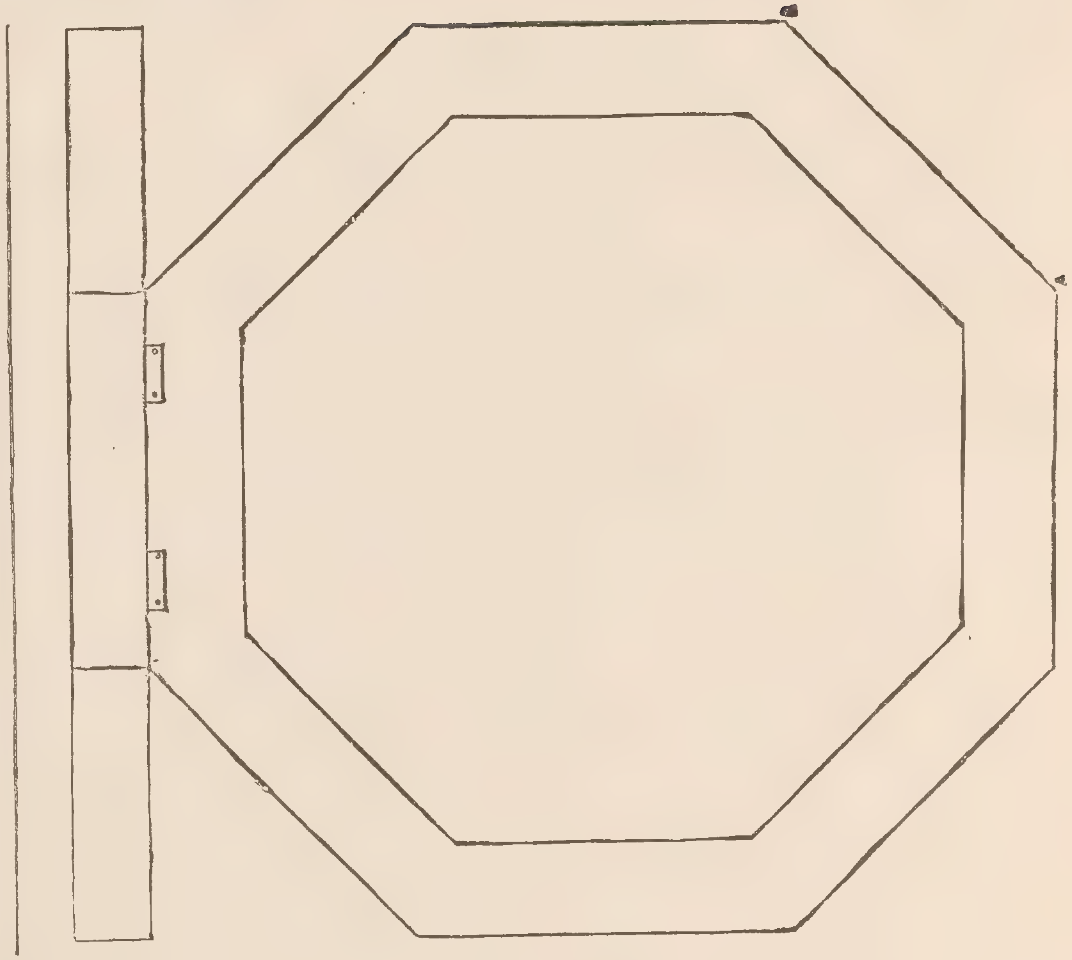
* The centre of vision is given, the eye of the spectator is to be **12′** by scale, distant from it, and **5′** above the ground-plane.

The lines used in working the problem *must* be shown.

The given figure is the plan of an octagonal box and lid. The box stands upon the ground-plane, and the lid is turned upward on its hinges into a vertical position. The point **A** (on the ground-plane) is **3′** from the picture-line and **2′** on the spectator's left hand, and the line **A B** is parallel to the picture-plane. The height of the box (exclusive of the lid) is **2′**. Give the perspective representation, showing only the visible lines of the object.

Scale, ½ inch to **1** foot.

*One hour and a half allowed.*



May, 1880.

Ex. 31.

* Distance of eye in front of the picture-plane, **10′**.

Height of eye above the ground-plane, **5′**.

The plan is given of a cupboard with the door open, the external height being **7′**. The point **A**, on the ground-plane, is to be **2′** on the left of the spectator and **2′** from the picture line, and the line **AB** is to recede towards right at an angle of **45°** with the picture-plane. The material is of the same thickness throughout. Give the perspective representation, including a shelf half-way between the bottom and top of the cupboard.

*N.B.—The solution of this problem is given in Plate* **M**.

May, 1880.

Ex. 32.

* The centre of vision is given. The distance of the eye from it is **12′**, and the height of the eye above the ground-plane is **5′**.

The accompanying diagram is the plan of a doorway through a wall with a semicircular step. The step is **1′** thick (rise) and the doorway is a rectangular opening **8′** in height. The dotted line **AB** lies upon the ground-plane and recedes towards right at **50°** with the picture plane. The point where the dotted line meets the semicircular edge of the step (point **A**) is **1′** on the spectator's left and **4′** from the picture-plane. Give the perspective representation of doorway and step. *(See Plate* **N**.)

May, 1880.

Ex. 33.

* Distance of eye in front of the picture-plane, **12′**.

Height of eye above the ground-plane, **5′**.

The elevation is given of a doorway through a wall and a step. The thickness of the wall is **18″**, the step is rectangular and projects **1′** from the wall. Give the perspective representation, the point **A** being **2′** on spectator's right and **1′** beyond the picture-plane, and the long edges of the step vanishing to left at an angle of **45°** with the picture-plane. (*See Plate* **O**.)

November, 1880.

Ex. 34.

* The centre of vision is given. The eye of the spectator is to be **12′**, by scale, distant from it, and **5′** above the ground-plane. Scale, $\frac{1}{2}''$ to **1′**.

The lines used in working the problem *must* be shown.

I. A right prism, the base of which is a regular hexagon of **6′** side, and having an axis **2′** long, lies upon the ground-plane on one of its hexagonal faces. The nearest angle on the ground plane is **3′** from the picture-line and **1′** on spectator's right, and *one* of the visible rectangular faces of the solid recedes towards left at an angle of **40°** with the picture-plane.

II. A right cone having an altitude of **7′** and a base of **8′** diameter stands upon the prism, and the two solids have their axes both in the same vertical line. Represent the cone and prism in perspective.

November, 1880.

Ex. 35.

* The centre of vision is given. The eye of the spectator is to be **12′** distant from it and **5′** above the ground-plane. Scale, $\frac{1}{2}''$ to **1′**.

The lines used in working the problem *must* be shown.

I. A circular slab, or short right cylinder, the diameter of which is **8′** and the thickness **1′** lies on the ground-plane. The centre of the circular face on which it rests is **6′** from the picture-line and **2′** on spectator's right.

II. A pyramid having a square base of **4′** side and an altitude of **6′** stands upon the slab. The sides of the base of the pyramid vanish towards right and left at angles with the picture-plane of **35°** and **55°** respectively. The axes of the two solids are in the same vertical line. Give the perspective representation of the slab and pyramid.

Ex. 36.

* The centre of vision is given. The eye of the spectator is to be **12′** distant from it and **5′** above the ground-plane. Scale $\frac{1}{2}''$ to **1′**.

The lines used in working the problem *must* be shown.

I. A right cylinder, **5′** diameter and **3′** long, stands upright upon the ground-plane, and the centre of the base upon which it rests is **4½′** from the picture-line and **2′** on right of the spectator.

II. A square slab of **6′** side and **1′** thick is laid upon the cylinder, its square faces being horizontal and its vertical faces receding from the picture towards right and left at angles of **35°** and **55°** respectively. The axes of the two solids are in the same straight line. Represent the cylinder and slab in perspective.

November, 1883.

Ex. 37.

* The centre of vision is given. The eye of the spectator is to be **12′**, by scale, distant from it, and **6′** above the ground-plane. Scale, $\frac{1}{2}''$ to **1′**.

The figures **A**, **B** and **C**, are the plans of three thick boards standing upon the ground-plane. The figure **D** is the plan of a right cylinder, also resting on the ground-plane, its axis being indicated by the dotted line. The height of each board is **8′**, and the vertical face **e f** is perpendicular to the picture-plane. The point **f** on the ground, is **3′** on left of **C V** and **2′** from the picture-line. (*See Plate* **S**.)

Ex. 38.

* The centre of vision is given. The eye of the spectator is to be **12′** distant from it, and **5′** above the ground-plane. Scale, $\frac{1}{2}''$ to **1′**.

I. A right prism, the bases of which are squares of **5′** side, and the axis of which is 3′, stands on its base on the ground-plane. Two of the vertical faces of the prism are perpendicular to the picture-plane, and the corner on the ground-plane which is nearest to the spectator, is **2′** on left and **4′** from the picture-line.

II. A right cylinder lies upon the prism, its axis is **11′** long, and the diameter of each base is **5′**. The axis of the cylinder recedes from the picture-plane at an angle of **45°** towards left, and the centres of the two solids are in the same vertical line.

October, 1889.

Ex. 40.

* The eye is to be **6′** by scale distant from the picture plane and **3′** above the ground plane. The scale to be used in working this problem is **1″** to **1′**.

The accompanying diagrams are drawn to *half-scale*, and represent a plan and two elevations of an iron garden-chair and a large flower-pot, both standing on the ground plane. Represent these two objects in perspective; the centre **A** of the base of the flower-pot is to be **1′ 6″** on the right of the spectator, and **1′** from the picture line, and the edge, **C D**, of the chair is to vanish towards the right at an angle of **50°** with the picture plane.

November, 1883.

Ex. 39.

* The centre of vision is given. The eye of the spectator is to be **12** distant, from it, and **5′** above the ground-plane. Scale, $\frac{1}{2}''$ to **1′**.

The elevations of a right cylinder and a right pyramid are given, the base of the latter is square. Represent these solids in perspective, the conditions, etc., being as follows:

Both objects rest upon the ground-plane. The point **C** in the nearer base of the cylinder is **3′** on left of spectator and **6′** from the picture-line. The axis of the cylinder is **10′** and recedes from the picture-plane at an angle of **40°** towards left.

The edge **A B** of the pyramid touches the side of the cylinder midway between its bases. (*See Plate* **T**.)

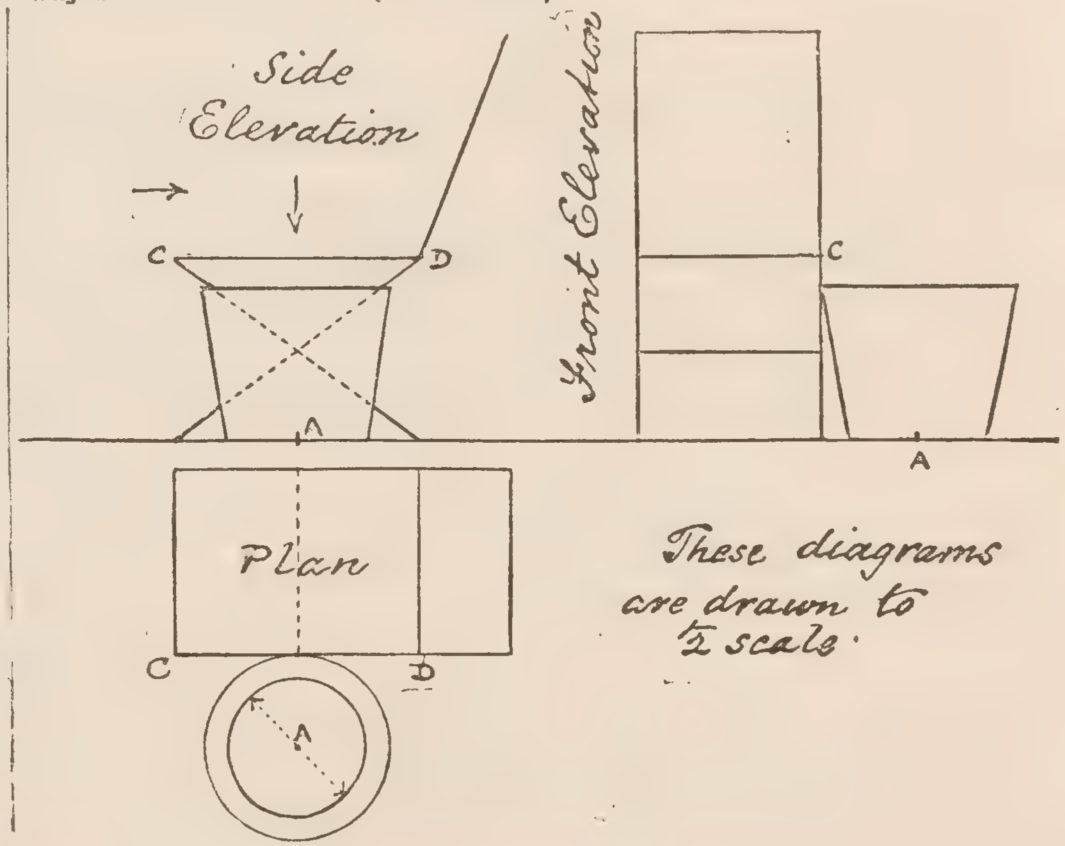
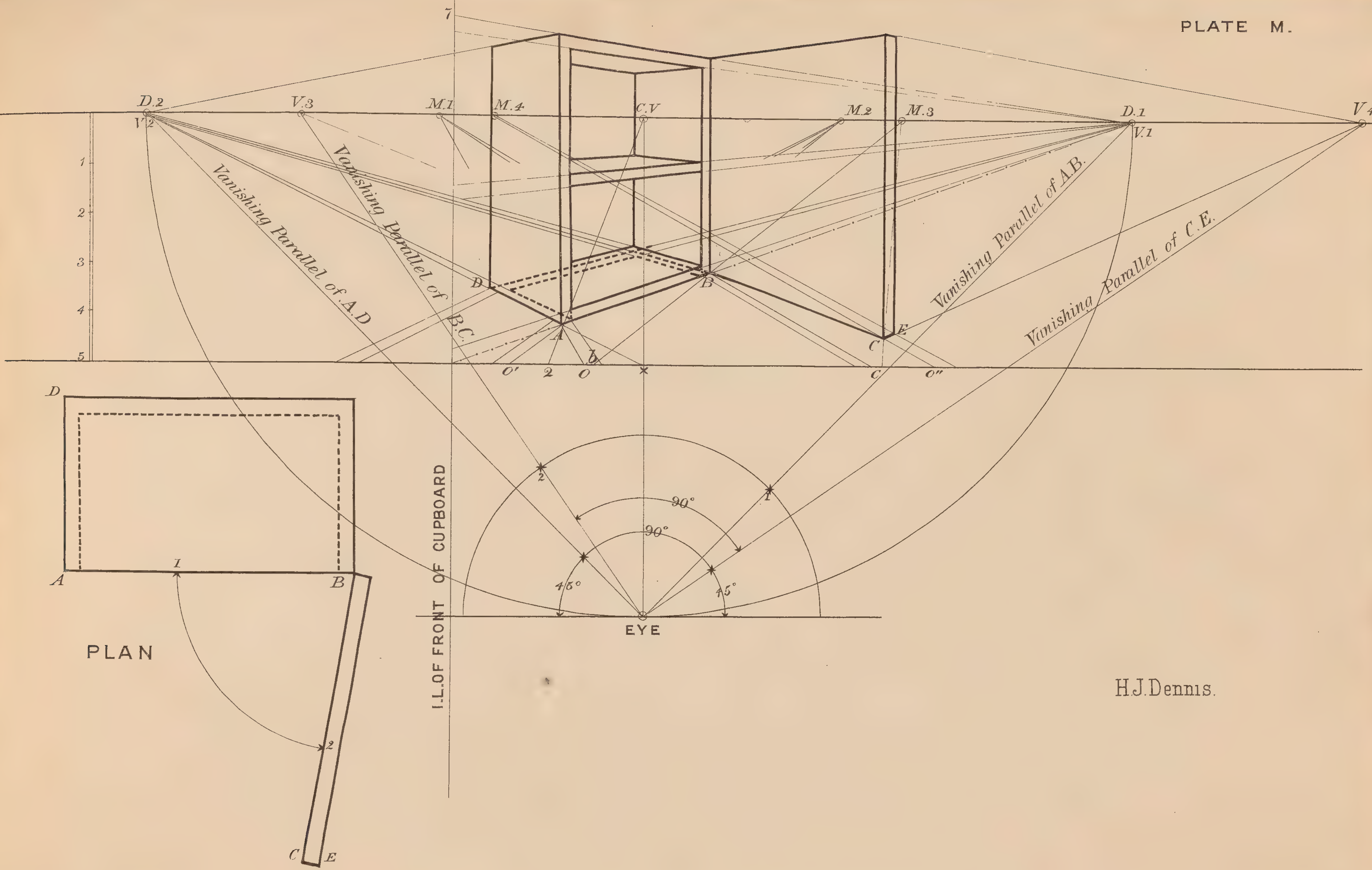
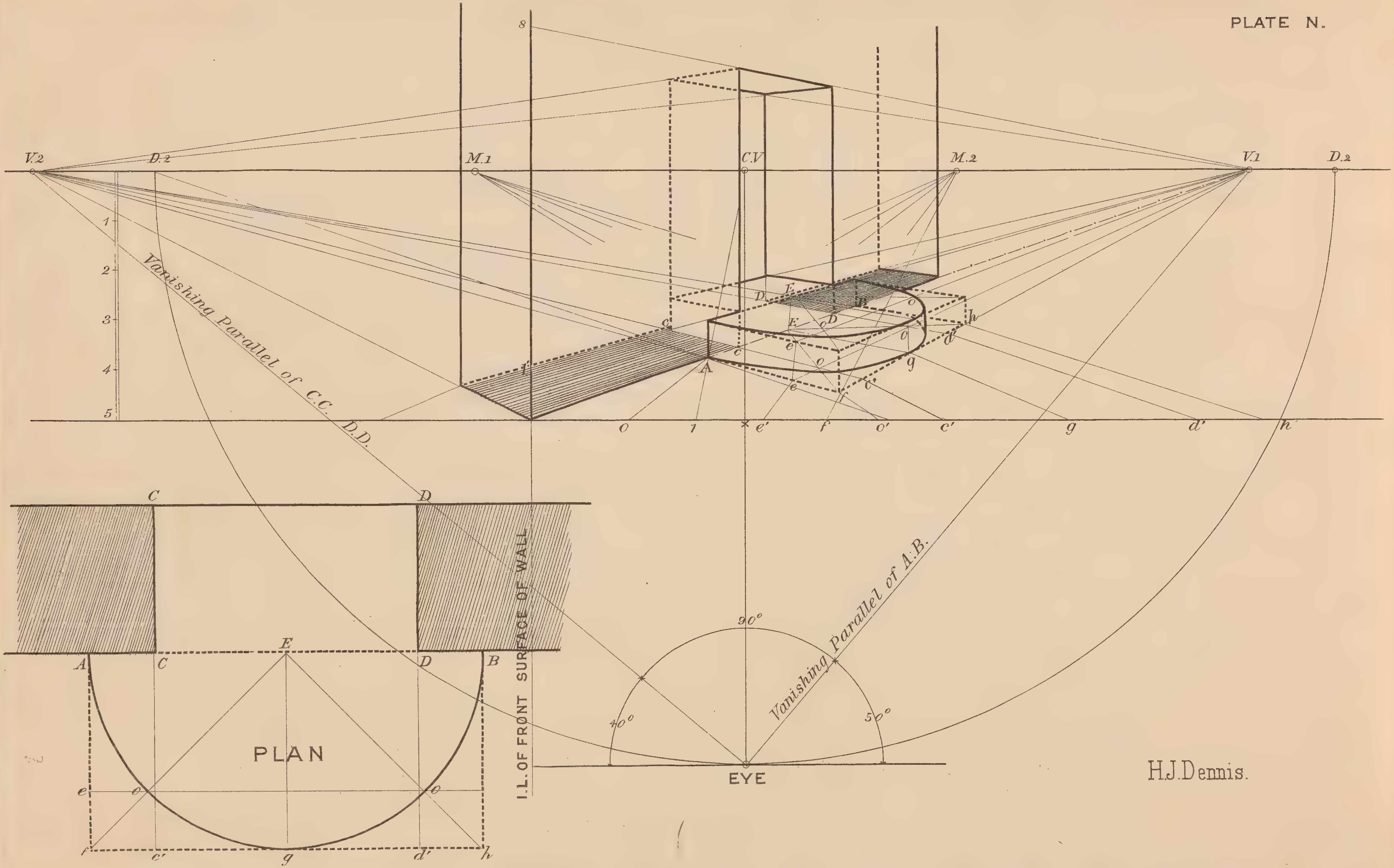


PLATE M.




H.J.Dennis.

PLATE N.




H.J.Dennis.

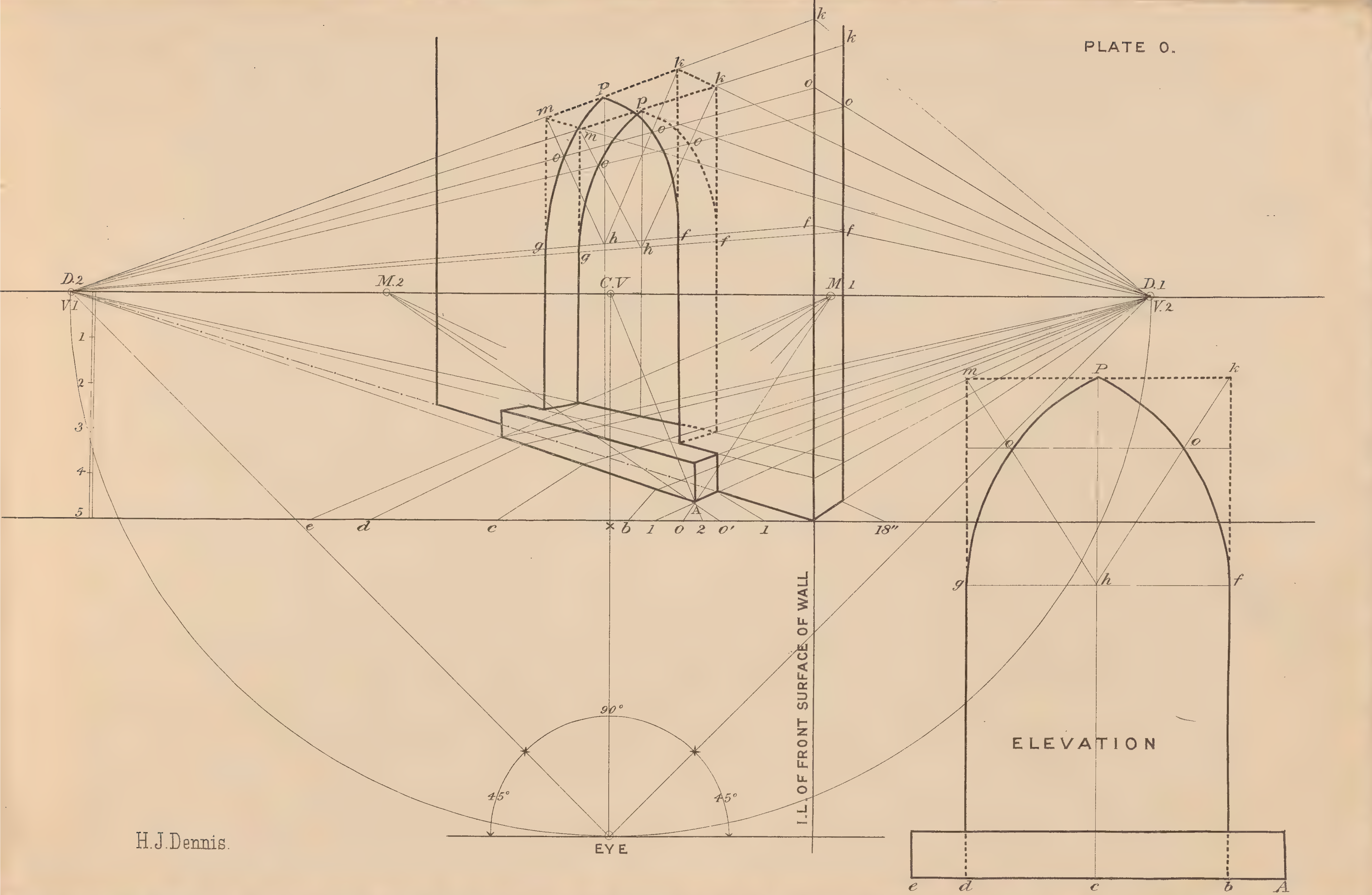
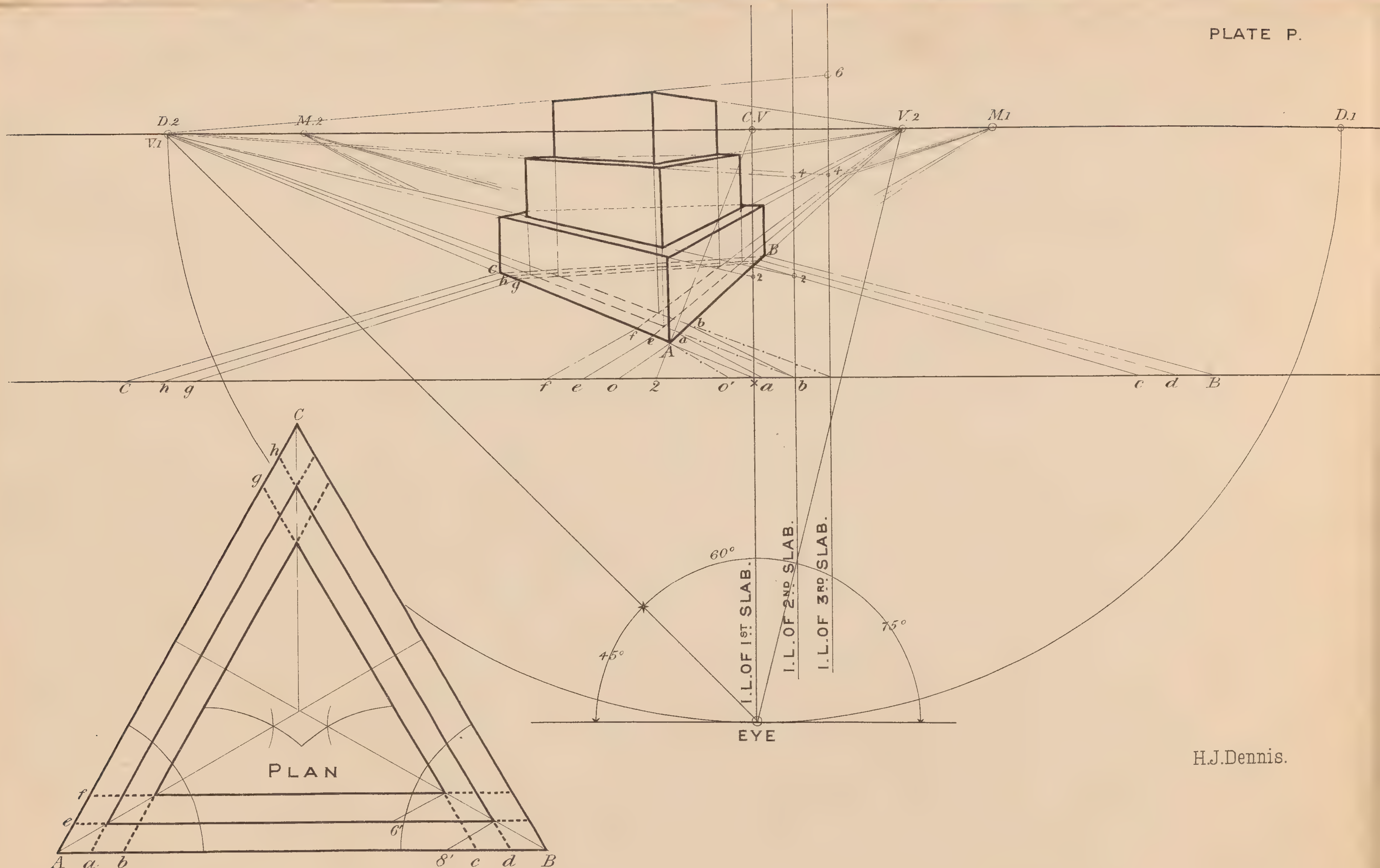


PLATE O.
D.2
V1
M.2
C.V
M 1
D.1
V.2
ELEVATION
I.L. OF FRONT SURFACE OF WALL
90°
45°
45°
EYE
18"
H.J.Dennis.


D.2
V.1
M.2
C.V
V.2
M.1
D.1
6
4
2
B
c
h g
f e
a
b
A
C h g f e o 2 o' a b c d B
C
h
g
PLAN
f
e
6'
A a b 8' c d B
60°
45°
75°
I.L. OF 1ST SLAB.
I.L. OF 2ND SLAB.
I.L. OF 3RD SLAB.
EYE

H.J.Dennis.

PLATE Q.

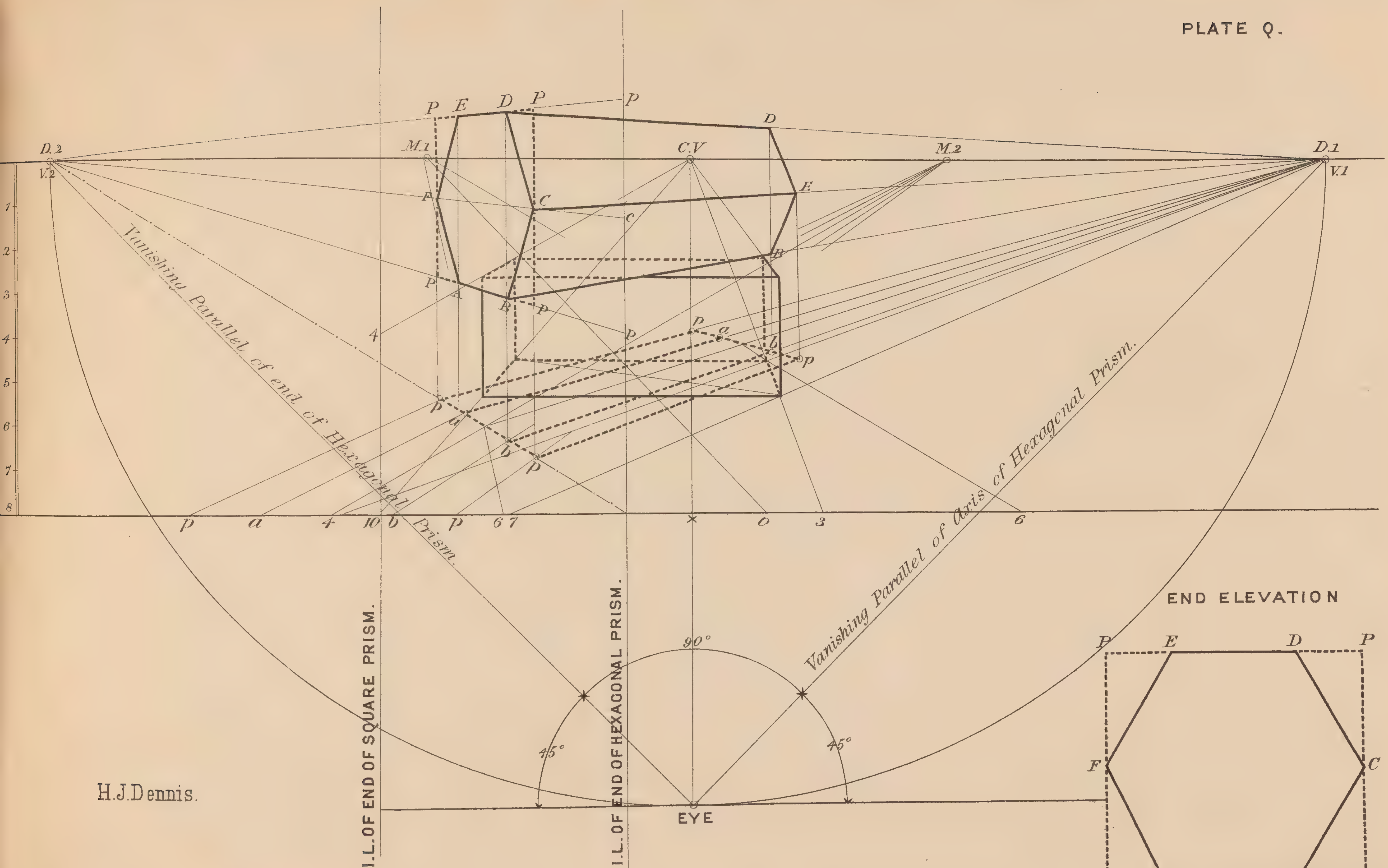



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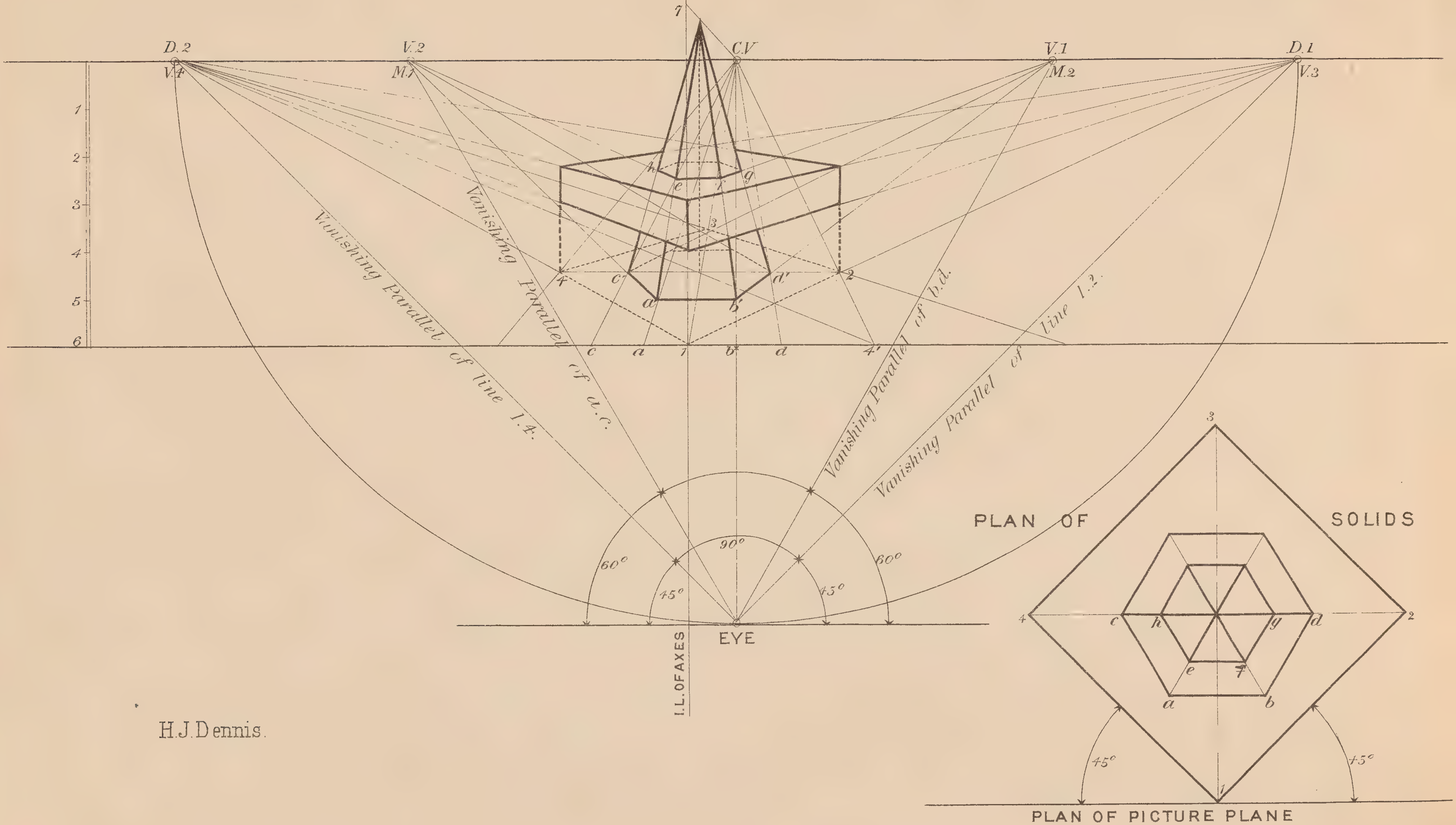
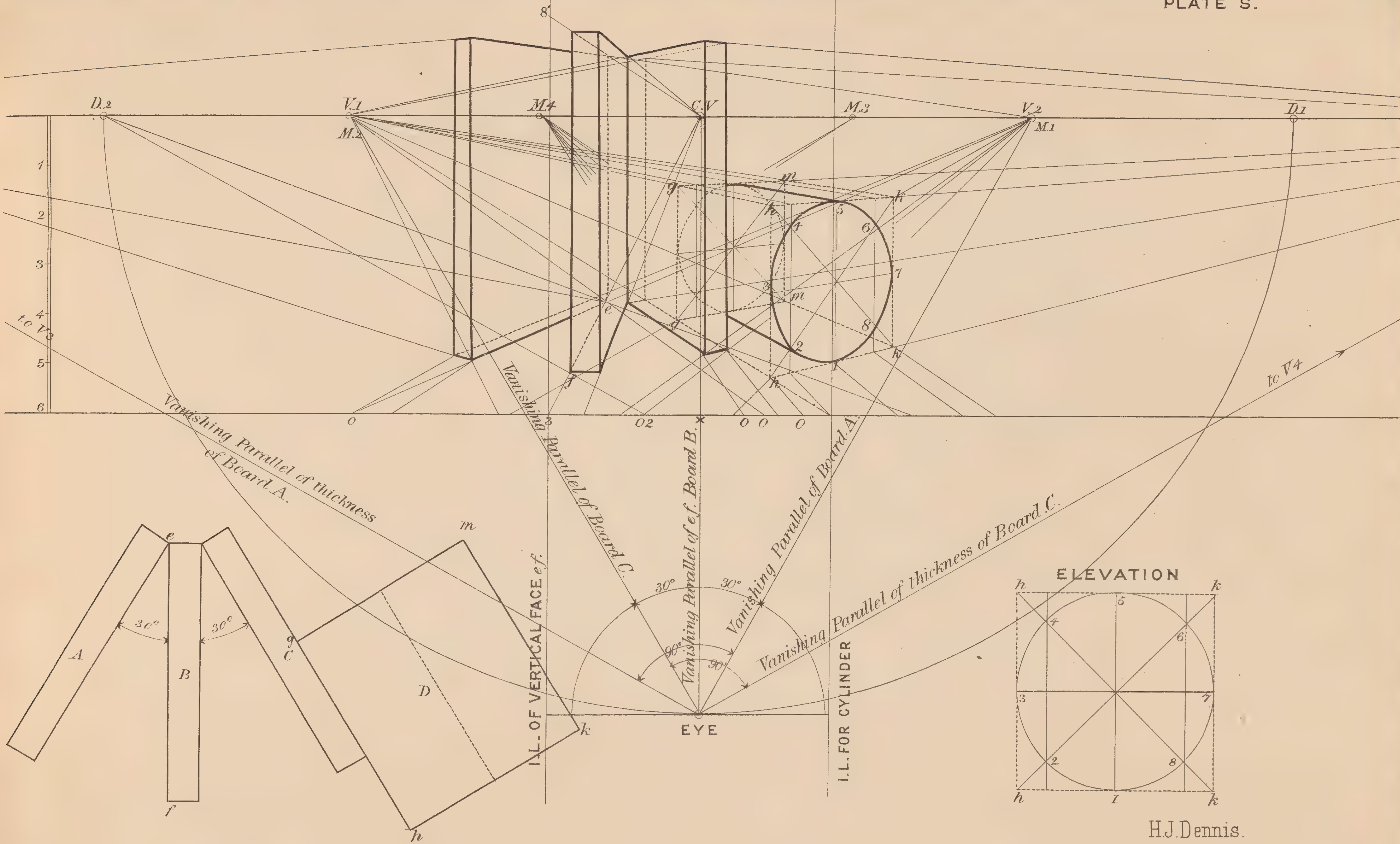
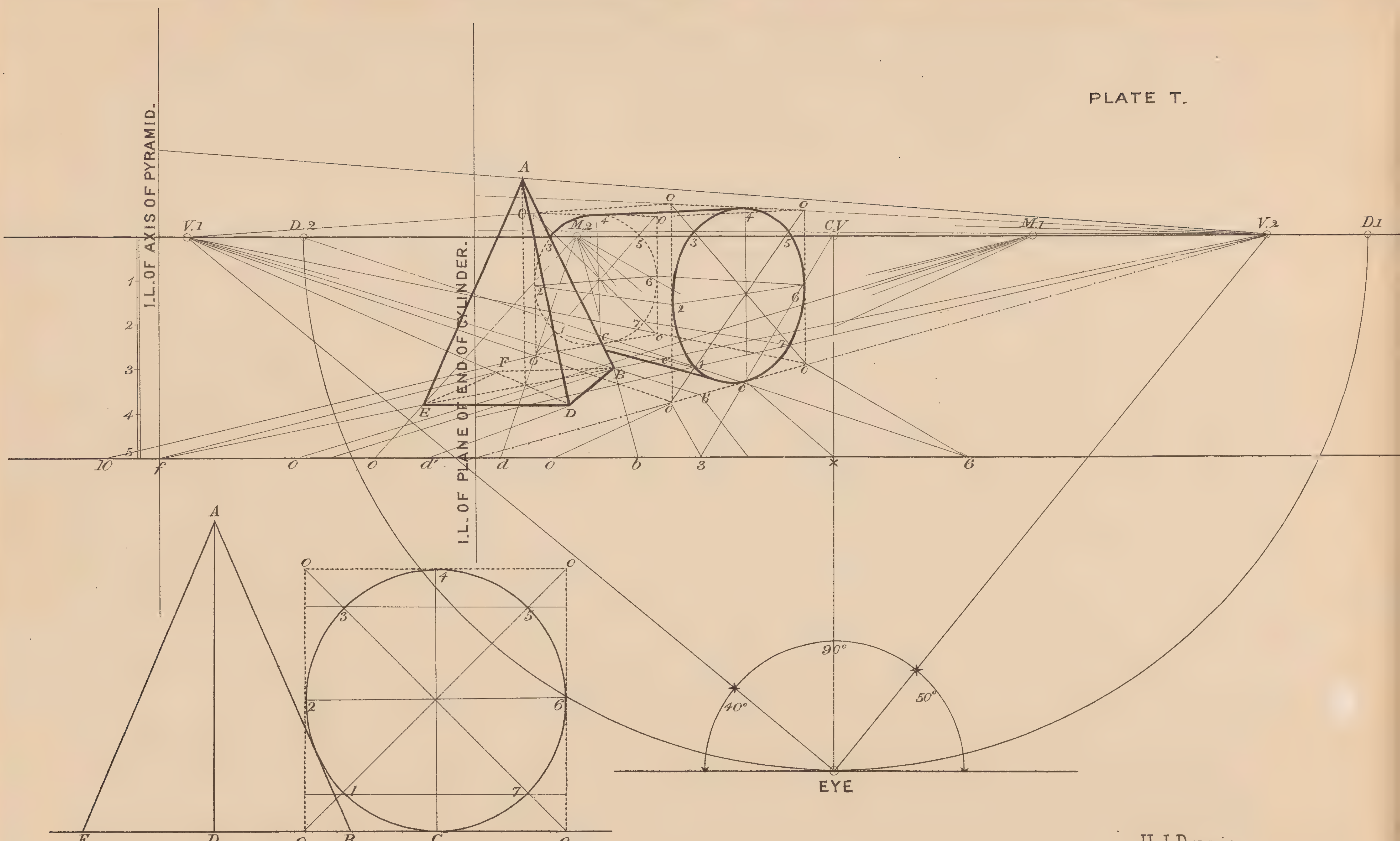
PLATE R.


D.2
V.4
V.2
M.1
C.V
V.1
M.2
D.1
V.3
Vanishing Parallel of line 1.4.
Vanishing Parallel of a.c.
Vanishing Parallel of b.d.
Vanishing Parallel of line 1.2.
60°
45°
90°
45°
60°
EYE
I.L. OF AXES
PLAN OF SOLIDS
45°
45°
PLAN OF PICTURE PLANE
H.J.Dennis.

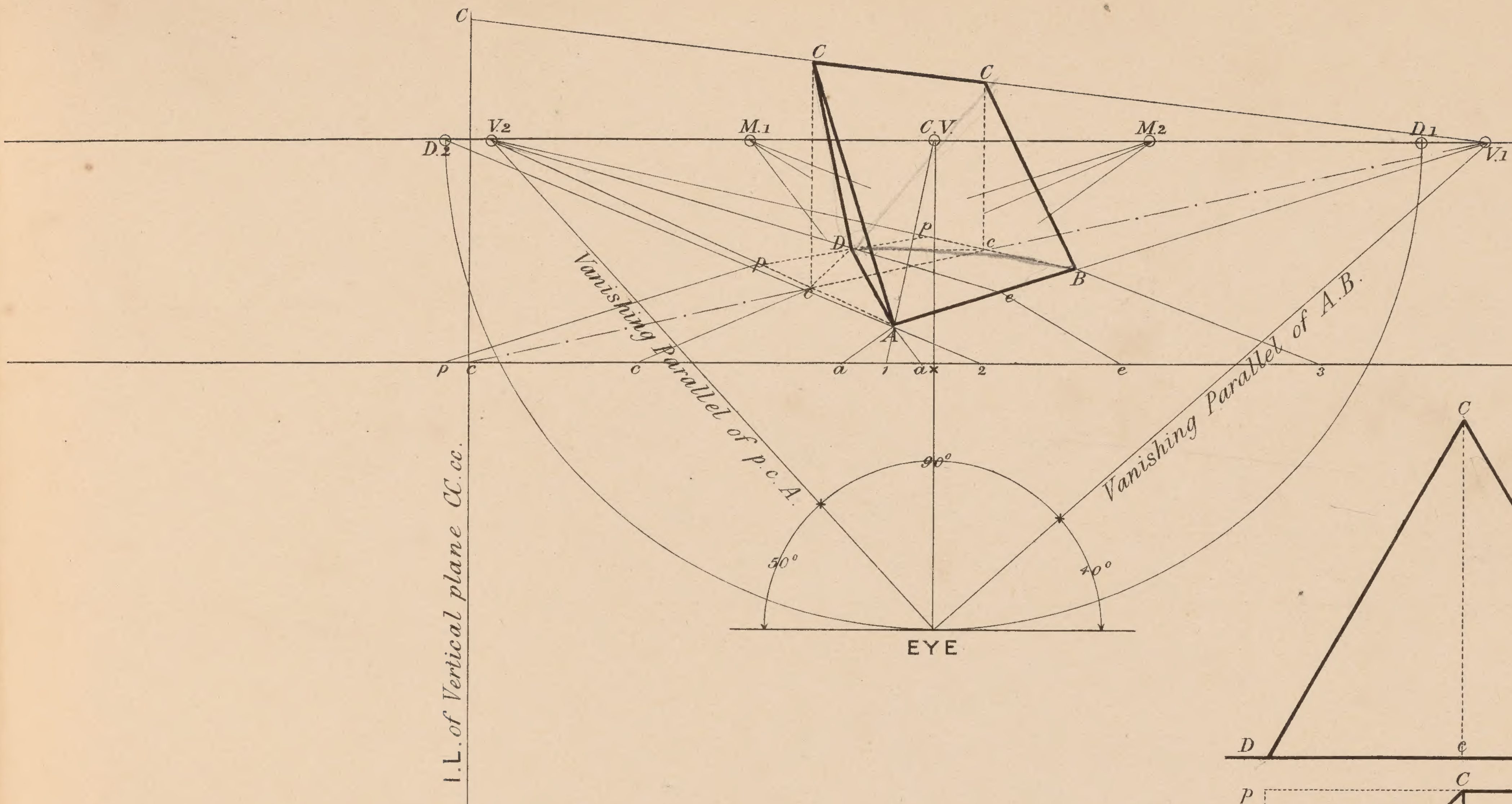

D.2
V.1
M.2
M.4
C.V
M.3
V.2
M.1
D.1
to V3
to V4
Vanishing Parallel of thickness of Board A.
Vanishing Parallel of Board C.
Vanishing Parallel of e.f. Board B.
Vanishing Parallel of Board A.
Vanishing Parallel of thickness of Board C.
I.L. OF VERTICAL FACE e.f.
I.L. FOR CYLINDER
EYE
30°
90°
A
B
C
D
ELEVATION

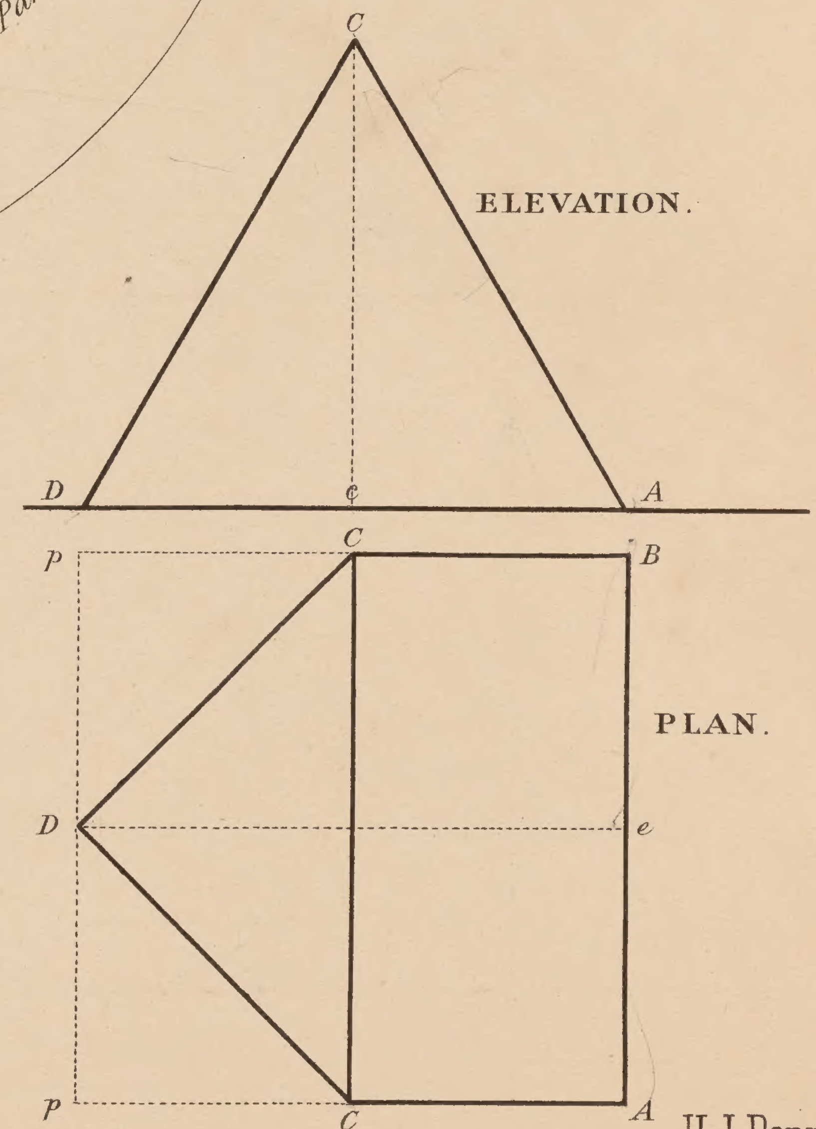
H.J.Dennis.

PLATE T.




H.J.Dennis.




### EXERCISE (ART DEPARTMENT, 1892).

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Distance of Eye, 11 in.

Height of Eye, 5 in.

The plan and elevation of a right pyramid with a square base are given. The solid rests upon the ground plane on one of its triangular faces. Required its perspective representation when corner A is 1 in. on left and 2 in. beyond the picture and the edge A B recedes towards right at an angle of 40° with the ground line.

H.J. Dennis.